

FRANC3D Training Workshop: Part 9

Crack Growth

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Workshop Agenda

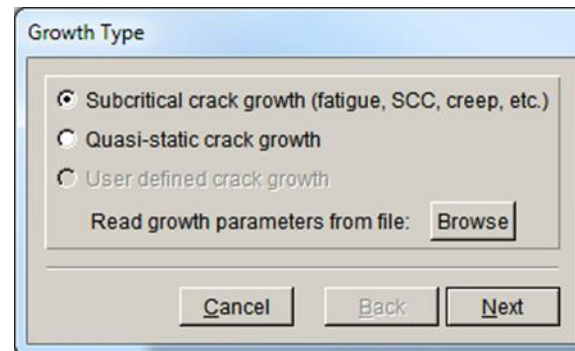
- Part 1: Introduction to Fatigue and Damage Tolerance
- Part 2: Introduction to Fracture Mechanics Analysis
- Part 3: Introduction to FRANC3D
- Part 4: FRANC3D User Interface
- Part 5: Finite Element (FE) Model Import
- Part 6: Crack Insertion
- Part 7: Static Crack Analysis & SIF Computation
- Part 8: SIFs from FE Analysis
- **Part 9: Crack Growth**
- Part 10: SIF History & Fatigue Life
- Part 11: Miscellaneous Topics

Crack Growth

- FRANC3D Crack Growth
- Crack Growth Prediction
- Crack Turning or Kinking
- Crack Relative Extension
- Crack Front Smoothing
- Subcritical Crack Growth
- Crack Growth Load Schedule
- Crack Growth Rate Models for Subcritical Crack Growth
- Quasi-Static Crack Growth
- User-Defined Crack Growth
- Crack Growth Stopping Criteria
- Fatigue Crack Growth Summary
- Demo: Crack Growth

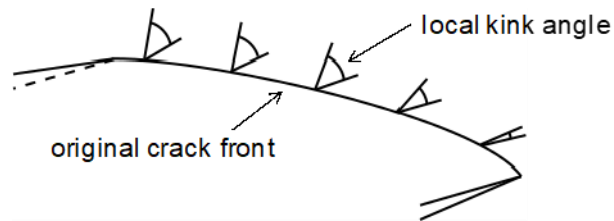
FRANC3D Crack Growth

- FRANC3D is used to perform the following:
 - Compute accurate SIFs
 - Predict how crack(s) will grow
 - Modify the FE mesh to incorporate the crack shape
- FRANC3D supports three types of crack growth prediction:
 - Subcritical
 - Quasi-static
 - User-defined

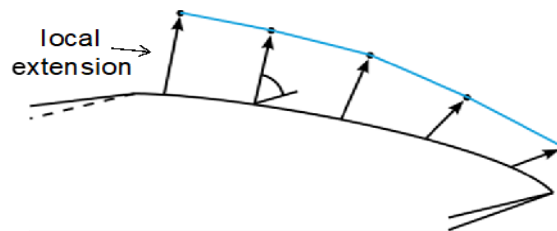


Crack Growth Prediction

- Computing crack growth is a three-step process:
 - Compute the kink angle for each node on the crack front, which gives direction



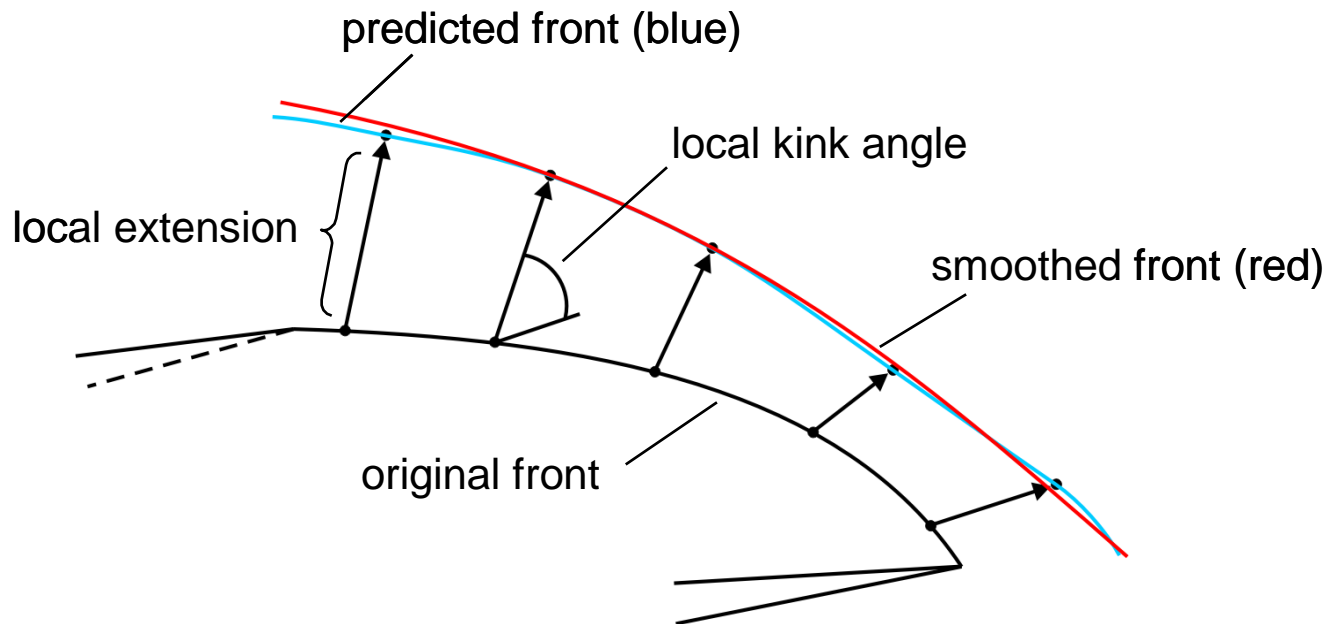
- Compute relative amount of local crack extension for each node on the crack front



SIFs are used to predict direction and relative growth.

Crack Growth Prediction

- Smooth the computed new crack front



SIFs are used to predict direction and relative growth.

Crack Turning or Kinking

Crack Propagation Kink Angle

- FRANC3D Supports
 - Following methods for computing the kink angle:
 - Maximum Tensile Stress
 - Maximum Shear Stress
 - Generalized Stress
 - Strain Energy Release Rate
 - Planar growth ($\theta=0.0$)
 - User defined model
 - Modifications for Anisotropic Toughness

Kink angle model

Kink angle model

Max tensile stress (MTS) $\max(K_I^r(\theta))$

Max shear stress (MSS) $\max\left(\sqrt{(\eta_{II} K_{II}^r(\theta))^2 + (\eta_{III} K_{III}^r(\theta))^2}\right)$

Generalized stress $\max(\text{MTS}, \text{MSS})$

Strain energy release rate $\max\left(K_I^r(\theta)^2 + (\eta_{II} K_{II}^r(\theta))^2 + (\eta_{III} K_{III}^r(\theta))^2\right)$

Planar $\theta = 0$

User defined model

Kink angle limit Maximum kink angle (deg):

Mixed mode eta factors

η_{II} : η_{III} :

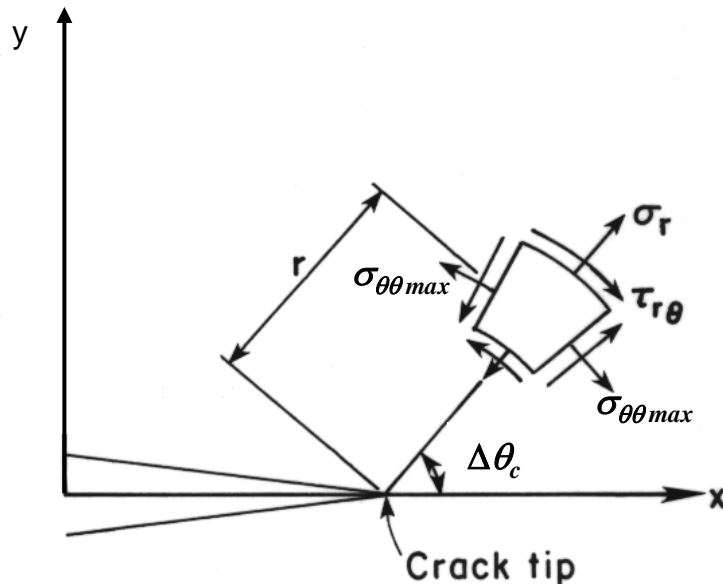
Crack growth resistance

Anisotropic toughness

Maximum Tensile Stress

$\sigma_{\theta\theta \max}$ theory:

- Crack kinks in direction normal to the maximum circumferential (hoop) stress; ignore higher order terms
- Default algorithm in FRANC3D



$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[K_I \cos^2 \frac{\theta}{2} - \frac{3}{2} K_{II} \sin \theta \right]$$

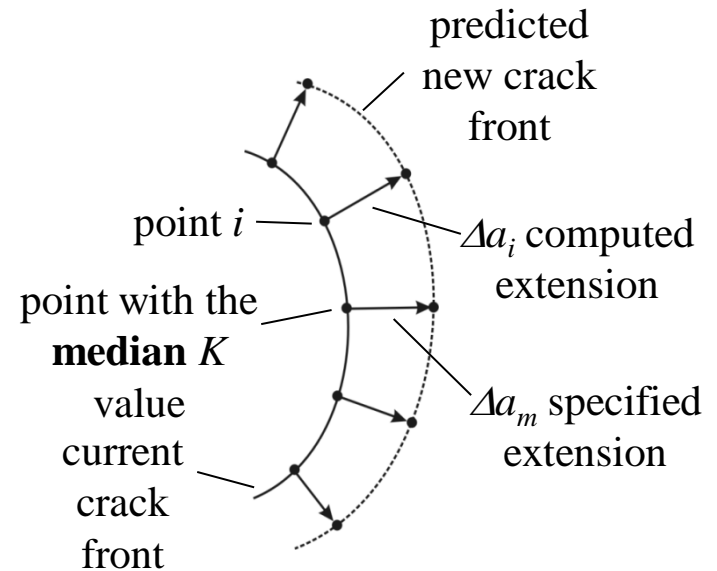
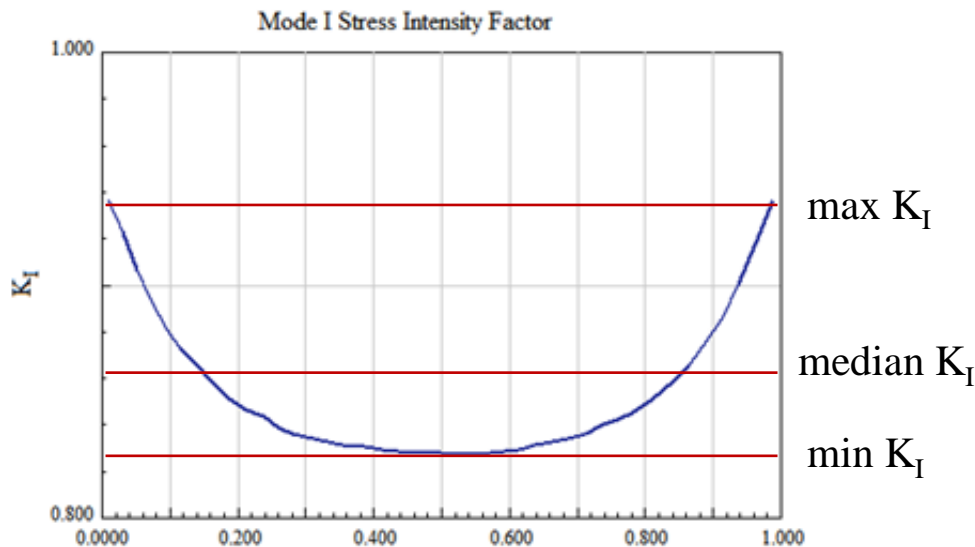
$$\text{or } \Delta\theta_c = 2 \tan^{-1} \left(\frac{1 - \sqrt{1 + 8(K_{II}/K_I)^2}}{4(K_{II}/K_I)} \right)$$

Crack Relative Extension

Crack Propagation Relative Extension

Simple quasi-static growth:

Extension at mid-side node- i = user-input-extension (or cycles) \times $\left[\frac{\text{SIF at } i}{\text{median SIF}} \right]^n$



Crack Front Smoothing

Crack Propagation Smoothing

Curve is extrapolated outside the model surface.

Specify median crack extension

Specifying extension is more stable than specifying cycles.

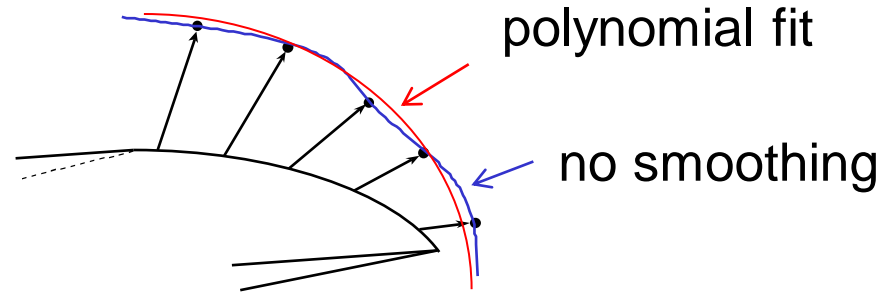
The screenshot displays the 'Crack Growth' software interface. The main window shows a 3D model of a crack front with a green dashed line representing the crack path. A red curve is overlaid on the crack front, labeled 'polynomial fit', and a blue curve is labeled 'no smoothing'. The interface includes a 'Crack Extension' section with a radio button selected for 'median extension' and a value of 0.2. The 'Front Fitting Options' section has several radio buttons, with 'Fixed Order Poly Through Points' selected. The 'polynomial order' is set to 3, and 'extrapolate (%)' is set to 3. The 'Display' section has 'Kink Angles' and 'Extension' selected. The interface also includes a 'Save' button, a 'Read' button, and a 'write .frit file' button.

Set crack front fitting options

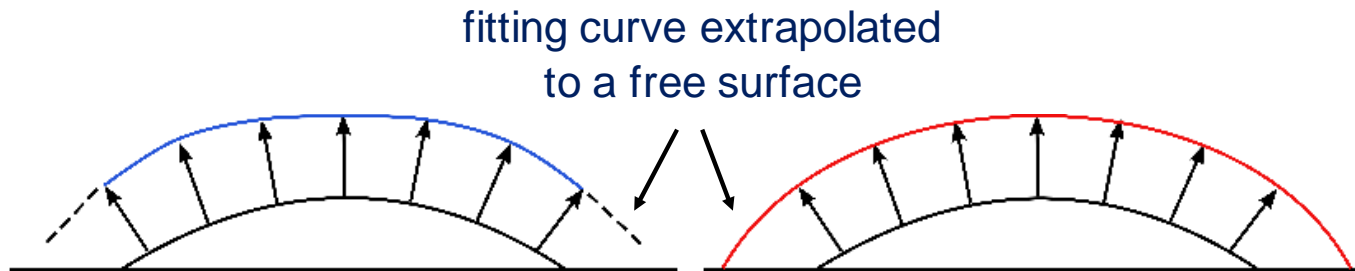
Use simplest fit that works.

Crack Propagation Smoothing

- The fitting is done for two reasons:
 - To smooth out numerical noise in the new front point predictions



- Smoothing yields an analytical curve that be used to interpolate or extrapolate to find the precise location where a crack front intersects a surface



Subcritical Crack Growth

Subcritical Crack Growth

- Fatigue or time dependent crack growth situations
- Stress intensity factors controlling crack growth are below the critical value for the material ($K < K_{crit}$)
- Crack growth is caused by fluctuating load levels (ΔK)
- Stress corrosion cracking and creep crack growth are examples of **time dependent crack growth**

Subcritical Crack Growth

Subcritical crack growth (crack extension) can be predicted considering cyclic fatigue, time dependent crack growth, or a combination.

$$\Delta a = \int_{\eta_0}^{\eta_1} \frac{da}{dN} (\Delta K, R, T) d\eta + \int_{\tau_0}^{\tau_1} \frac{da}{dt} (K, T) d\tau$$

where:

$$\Delta K = K_{\max} - K_{\min} \quad R = K_{\min} / K_{\max}$$

Δa = Crack extension,

da/dN = Crack growth rate due to load cycles

da/dt = Crack growth rate due to time dependent mechanisms

η_0, η_1 = Load cycle counts at the beginning and end of the crack extension step

τ_0, τ_1 = Time at the beginning and end of the step

T = Temperature at a location on the crack front

Crack Growth Rate Models

Cyclic Load Models:

- Paris Equation
- Bilinear Paris Equation
- Hyperbolic Sine Equation
- Sigmoidal Equation
- NASGRO Equation
- Tabular Data

Stress Ratio Models:

- No Stress Ratio Effects
- Walker Equation
- Newman Closure Equation
- Tabular Data

Time Dependent Models:

- Power Law Equation
- Terachi SCC Equation[†]
- Tabular Data

Growth models can be temperature dependent or temperature independent

[†] T. Terachi, *et. al.* “SCC growth behaviors of austenitic stainless steels in simulated PWR primary water,” *Journal of Nuclear Materials* 426 (2012) 59–70

Fatigue Growth Rate Models

- All fatigue growth rate models are a function of stress intensity factor range, ΔK . This is the difference between the maximum stress intensity factor in a load cycle, denoted K_{\max} , and the minimum value, K_{\min} (that is, $\Delta K = K_{\max} - K_{\min}$)
- The fatigue crack growth rate may also be a function of the mean stress intensity factor, typically characterized by the stress ratio, R , where $R = K_{\min} / K_{\max}$
- Fatigue crack growth rates can be specified to be temperature-dependent or temperature-independent

Time Dependent Growth Rate Models

- Time dependent crack growth rates, da/dt , are functions of crack-front stresses, temperature, and possibly other material properties
- Time dependent growth includes creep crack growth[†], stress corrosion cracking, and frequency dependent environmentally assisted crack growth.

[†] currently creep crack growth is K dependent only, C^* is not yet implemented.

FRANC3D Subcritical Crack Growth Procedure

Growth Type

Subcritical crack growth (fatigue, SCC, creep, etc.)
 Quasi-static crack growth
 User defined crack growth

Read growth parameters from file:

Kink angle model

Kink angle model

Max tensile stress (MTS) $\max(K_I^r(\theta))$
 Max shear stress (MSS) $\max\left(\sqrt{(\eta_{II}K_{II}^r(\theta))^2 + (\eta_{III}K_{III}^r(\theta))^2}\right)$
 Generalized stress $\max(\text{MTS, MSS})$
 Strain energy release rate $\max\left(K_I^r(\theta)^2 + (\eta_{II}K_{II}^r(\theta))^2 + (\eta_{III}K_{III}^r(\theta))^2\right)$
 Planar $\theta = 0$
 User defined model
 Kink angle limit Maximum kink angle (deg):

Mixed mode eta factors

η_{II} : η_{III} :

Crack growth resistance

Anisotropic toughness

Subcritical Growth Parameters

Units used in FE model

stress: MPa length: mm temp: C time: sec

Crack Growth Load Schedule

Crack Growth Rate Model

Mixed-mode equivalent K

$K^{\text{equiv}} = K_I$ $K^{\text{equiv}} = \sqrt{K_I^2 + (\gamma_{II}K_{II})^2 + (\gamma_{III}K_{III})^2}$
 $K^{\text{equiv}} = K_{\text{RSS}}$ γ_{II} : γ_{III} :
 sign: from K_I from K_{II} from K_{III} always positive always negative

Effective Delta K

$\Delta K_{\text{eff}} = K_{\text{max}} - K_{\text{min}}$ $\Delta K_{\text{eff}} = K_{\text{max}} - \max(K_{\text{min}}, 0)$
 Use load interaction model

Integration Options

Accelerated counting Constant K for time integration

Dynamic Pairing Metric

$\Delta K_{\text{eff}}^{\text{equiv}}$ da/dN $\Delta CTOB$

Subcritical Crack Growth Parameters

Subcritical crack growth parameters

- Units used in FE model
- Crack growth load schedule
- Crack growth rate model
- Mixed-mode equivalent K
- Effective delta K
- Integration options
- Dynamic Pairing Metric

The screenshot shows a dialog box titled "Subcritical Growth Parameters" with several sections for configuring crack growth analysis. The "Units used in FE model" section shows "stress: MPa length: mm temp: C time: sec" with a "Change" button. The "Crack Growth Load Schedule" section has a text input field and buttons for "New Schedule", "Read From File", "Wizard", "View/Edit", and "Save To File". The "Crack Growth Rate Model" section has a text input field and buttons for "New Model", "Read From File", "View/Edit da/dN", "View/Edit da/dt", and "Save To File". The "Mixed-mode equivalent K" section offers two radio button options: $K^{equiv} = K_I$ (selected) and $K^{equiv} = \sqrt{K_I^2 + (\gamma_{II} K_{II})^2 + (\gamma_{III} K_{III})^2}$. Below these are input fields for γ_{II} and γ_{III} , both set to 0. The "sign:" section has radio buttons for "from K_I ", "from K_{II} ", "from K_{III} ", "always positive", and "always negative". The "Effective Delta K" section has two radio button options: $\Delta K_{eff} = K_{max} - K_{min}$ (selected) and $\Delta K_{eff} = K_{max} - \max(K_{min}, 0)$. There is a checkbox for "Use load interaction model" and a "Set/Edit Parameters" button. The "Integration Options" section has checkboxes for "Accelerated counting" (checked) and "Constant K for time integration". The "Dynamic Pairing Metric" section has radio buttons for ΔK_{eff}^{equiv} (selected), da/dN , and $\Delta CROD$. At the bottom are "Cancel", "Back", and "Finish" buttons.

Units Used in FE model

- User must set the units used in the FE model
- Either SI or US units can be chosen
- FRANC3D needs to match the FE units with crack growth rate units, which can be different
- FRANC3D does unit conversions automatically

Units

Set Units:

SI units

Length: mm m other

Stress: MPa Pa other

Temperature: C K other

US customary units

Length: inch other

Stress: psi ksi other

Temperature: F other

Units of time:

sec min hour day year other

Cancel Accept

Crack Growth Load Schedule

Finite Element Results:

load step 1:

node displacements
(node temperatures)

load step 2:

node displacements
(node temperatures)

...

load step n:

node displacements
(node temperatures)

Fatigue Growth Requirements:

load cycle 1

K_{max} , K_{min} , temp, period

load cycle 2

K_{max} , K_{min} , temp, period

load cycle 3

K_{max} , K_{min} , temp, period

...

load cycle m

K_{max} , K_{min} , temp, period



Crack Growth Load Schedule

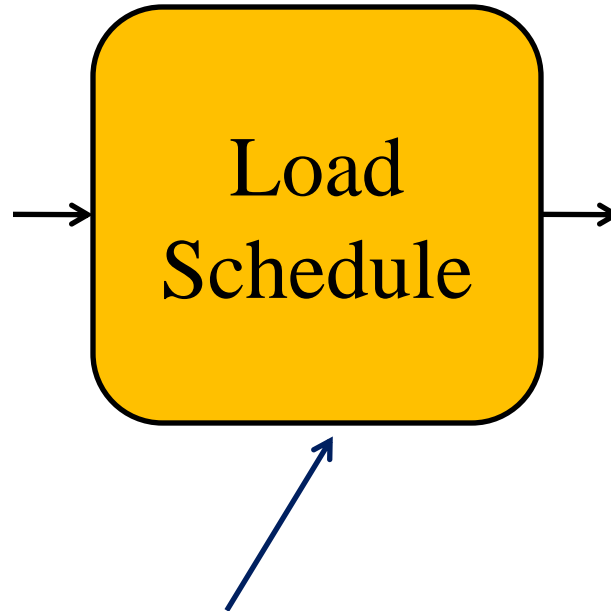
Finite Element Results:

load step 1:
node displacements
(node temperatures)

load step 2:
node displacements
(node temperatures)

...

load step n:
node displacements
(node temperatures)



Fatigue Growth Requirements:

load cycle 1
Kmax, Kmin, temp, period

load cycle 2
Kmax, Kmin, temp, period

load cycle 3
Kmax, Kmin, temp, period

...

load cycle m
Kmax, Kmin, temp, period

The fatigue “load schedule” defines a mapping from FE results to a sequence of fatigue load steps that are used to compute crack growth rates

Crack Growth Load Schedule

- Load Schedule
 - Used define a sequence of load cycles and/or hold times that will be applied to a finite element model for the purpose of predicting subcritical crack growth
 - User specified table of “load events”
 - Load Schedule:
 - Load Event 1
 - Load Event 2
 - Load Event 3
 - ...
 - Load Event n
- Users in different industries specify different types of fatigue load sequences, so load schedules are flexible (thus complex), but a user is not likely to need all capabilities in a single analysis.

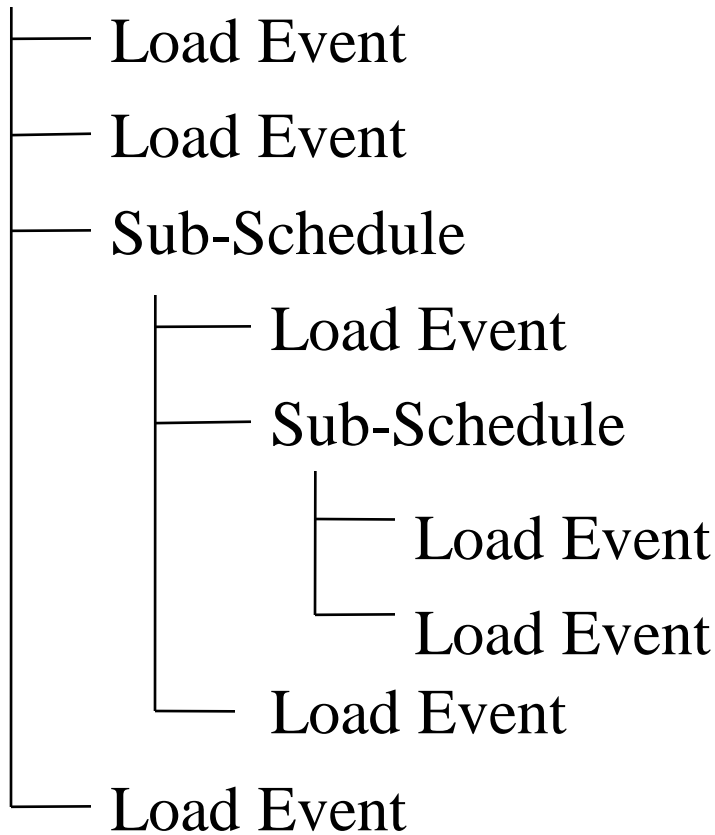
Crack Growth Load Schedule

- A load schedule can include one or many load steps
- For a jet engine component, a user might define a set of load steps where each step has a different temperature distribution and a different angular velocity; each load step represents a different time point in a typical flight (e.g., ground idle, takeoff/climb, cruise, maneuver, descent, reverse thrust)
- In FRANC3D, a user constructs a ‘Load Sequence’ that describes a typical mission or flight. For each event in the sequence, an optional load and temperature multiplier can be applied, giving a sequence table such as:

<u>Load Step</u>	<u>Load Mult</u>	<u>Temp Mult</u>	<u>Comment</u>
1	1.0	0.5	Ground idle (cold)
3	1.0	1.0	Climb
2	1.0	1.0	Initial cruise
4	0.8	1.0	Maneuver 1
2	1.1	1.0	Second cruise
4	1.2	1.1	Maneuver 2
2	1.0	1.0	Third cruise
5	1.0	1.0	Descent
1	1.0	1.0	Ground idle (hot)

Crack Growth Load Schedule

Load Schedule



- A load schedule event is a recursive definition of a (sub) load schedule, which allows load schedules to be defined with a hierarchical tree-like structure.
- This, combined with repeat counts for a sub-schedule, can simplify defining and storing a complex schedule.

Crack Growth Load Schedule

- Select the New Schedule button to display the Load Schedule dialog

Subcritical Growth Parameters

Units used in FE model
stress: MPa length: mm temp: C time: sec Change

Crack Growth Load Schedule
New Schedule Read From File Wizard View/Edit Save To File

Crack Growth Rate Model
New Model Read From File View/Edit da/dN View/Edit da/dt Save To File

Mixed-mode equivalent K
 $K^{\text{equiv}} = K_I$ $K^{\text{equiv}} = \sqrt{K_I^2 + (\gamma_{II} K_{II})^2 + (\gamma_{III} K_{III})^2}$
 $K^{\text{equiv}} = K_{\text{RSS}}$ γ_{II} : γ_{III} :
sign: from K_I from K_{II} from K_{III} always positive always negative

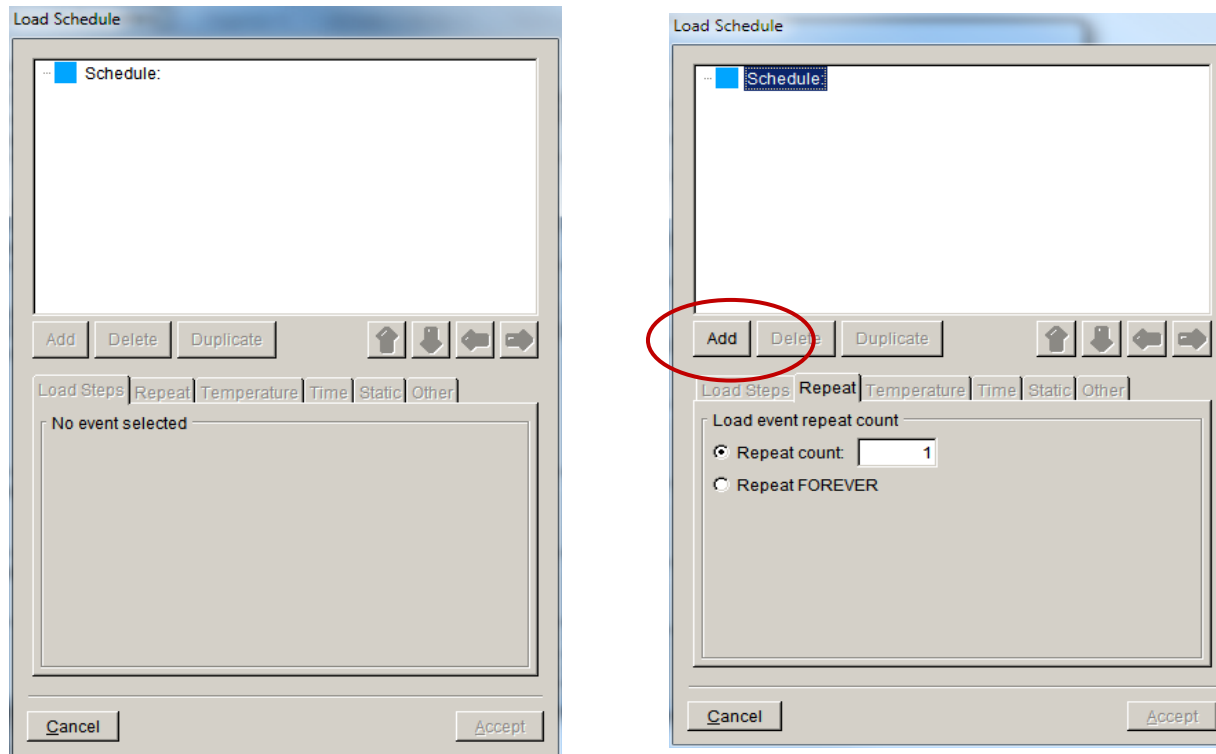
Effective Delta K
 $\Delta K_{\text{eff}} = K_{\text{max}} - K_{\text{min}}$ $\Delta K_{\text{eff}} = K_{\text{max}} - \max(K_{\text{min}}, 0)$
 Use load interaction model Set/Edit Parameters

Integration Options
 Accelerated counting Constant K for time integration

Dynamic Pairing Metric
 $\Delta K_{\text{eff}}^{\text{equiv}}$ da/dN ΔCTD

Cancel Back Finish

Crack Growth Load Schedule



- Load schedule is comprised of a collection of load events
- Load events are organized in a tree-like structure
- Select the **Add** button to display the Event Type dialog

Load Events

- FRANC3D supports the following load event types:
 - Simple Cyclic
 - Non-Proportional Cyclic
 - Transient
 - Spectrum
 - Hold – hold-time only with no cyclic loads
 - Dynamic Pairing – examines all SIFs and dynamically creates max-min pairs
 - Schedule – a load “sub-schedule”



Event type

Event Type:

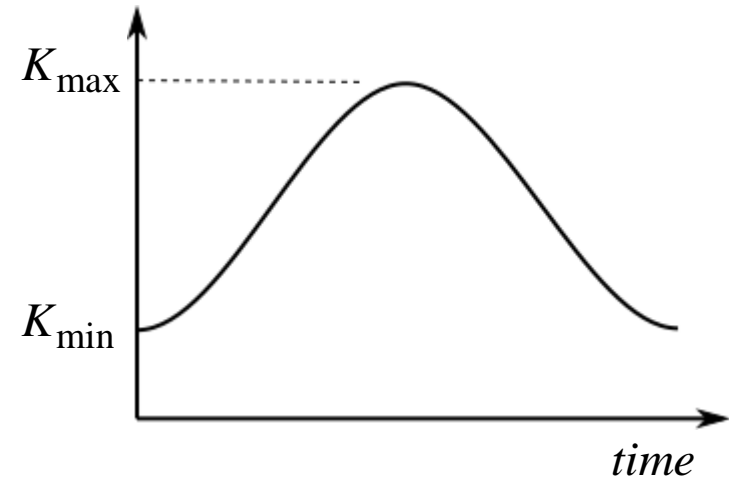
- Simple Cyclic
- Non-Proportional Cyclic
- Transient
- Spectrum
- Hold Event
- Dynamic Pairing
- Schedule

Cancel Accept

Load Events

- **Simple Cyclic**

- K_{max} from one FE load step or the sum of a number of FE load steps (with optional multipliers).
- $K_{min} = \text{zero or } R(K_{max})$, where R is stress ratio
- User can specify:
 - Stress ratio (R)
 - Load step (and substep if there is more than one) along with load and temperature multipliers
 - Temperature offset



Load Steps	Repeat	Temperature	Time	Static	Other
Simple cyclic load event					
Stress ratio, R: <input type="text" value="0"/>					
Step	Sub	Load Mult	Temp Mult	Temp Offset	
<input type="text" value="1"/>	<input type="text" value="0"/>	<input type="text" value="1"/>	<input type="text" value="1"/>	<input type="text" value="0"/>	<input type="button" value="Edit"/>

Load Events

- **Non-Proportional Cyclic:**
 - Cycles where Kmin and Kmax come from different FE load steps
 - User can specify for Kmin and Kmax:
 - Load step (and substep if there is more than one)
 - Load multipliers
 - Temperature multipliers
 - Temperature offset

Load Steps | Repeat | Temperature | Time | Static | Other

Nonproportional cyclic load event

K max load step:

No load step selected

K min load step:

No load step selected

Select Load Step

Load Step:

Load Sub-Step:

Load Multiplier:

Temperature Multiplier:

Temperature Offset:

Load Schedule

Schedule:

- NonPropCyclic: Kmax(),Kmin()

Load Steps | Repeat | Temperature | Time | Static | Other

Nonproportional cyclic load event

K max load step:

Step	Sub	Load Mult	Temp Mult	Temp Offset	Edit
1	-	1	1	0	<input type="button" value="Edit"/>

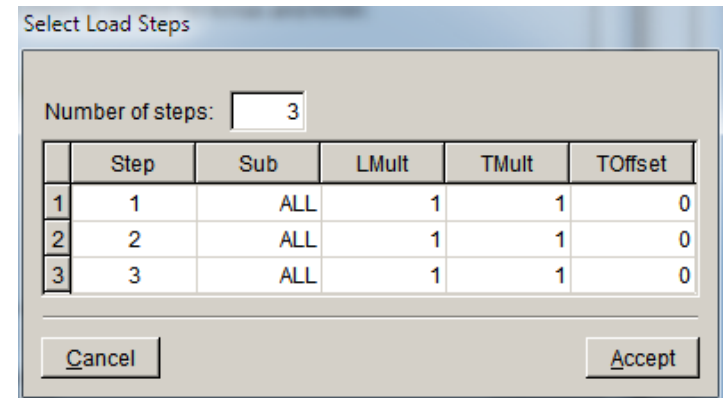
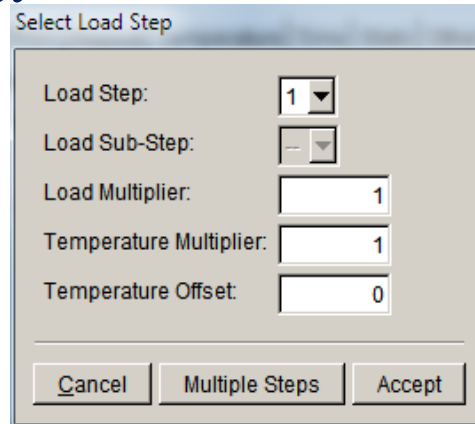
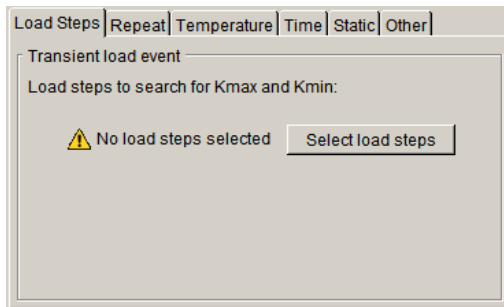
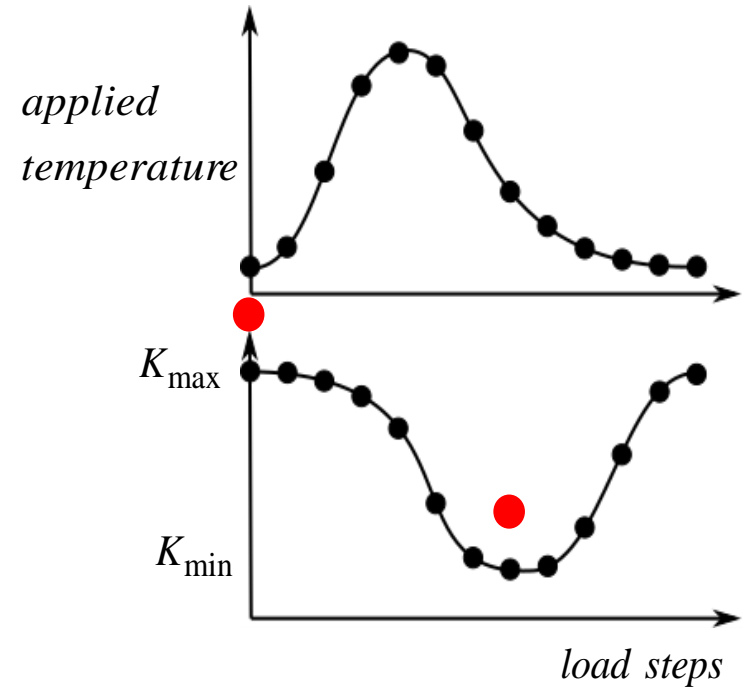
K min load step:

Step	Sub	Load Mult	Temp Mult	Temp Offset	Edit
2	-	1	1	0	<input type="button" value="Edit"/>

R and ΔK values are computed

Load Events

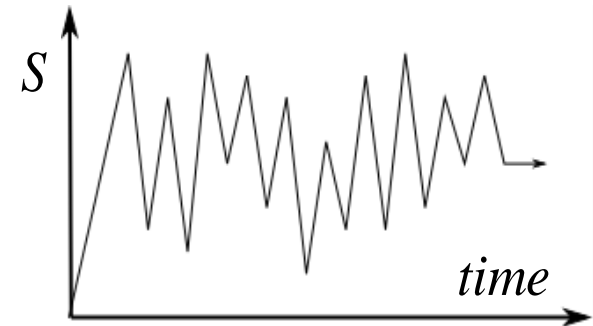
- **Transient:**
 - FRANC3D looks through all load steps (times) in the transient to find K_{max} and K_{min}
 - Different nodes on a crack front might get K_{max} and K_{min} from different times
 - The user must specify the load steps (and substeps if more than one) along with load and temperature multipliers and temperature offset



Load Events

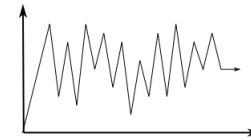
- **Spectrum loading:**

- For one load step, a load spectrum gives sequence of (S_{\max} , S_{\min}) pairs that are multiplied by K for the one step to get series of (K_{\max} , K_{\min}) pairs.
- For multiple non-proportional load steps, each load step has its own independent spectrum and K values for each cycle are:

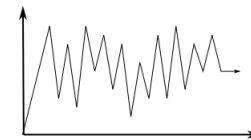


$$K_{\max} = \sum_{i=1}^n K_{\max}^{(i)}$$

$$K_{\min} = \sum_{i=1}^n K_{\min}^{(i)}$$

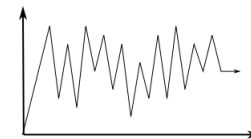


$$(S_{\max}^{(1)}, S_{\min}^{(1)}) \rightarrow (K_{\max}^{(1)}, K_{\min}^{(1)})$$



$$(S_{\max}^{(2)}, S_{\min}^{(2)}) \rightarrow (K_{\max}^{(2)}, K_{\min}^{(2)})$$

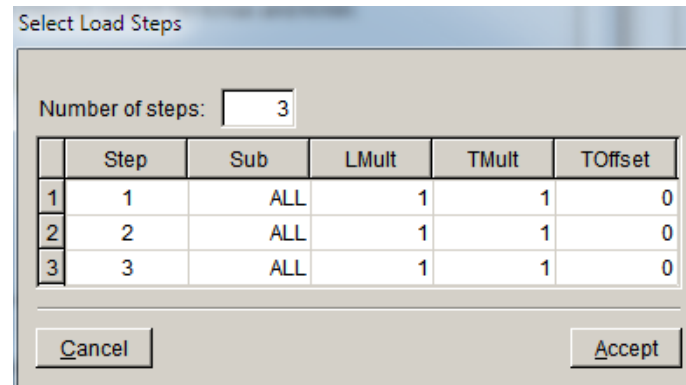
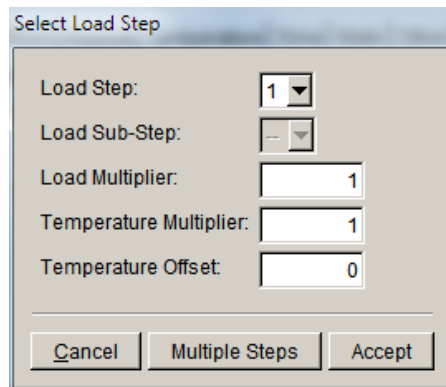
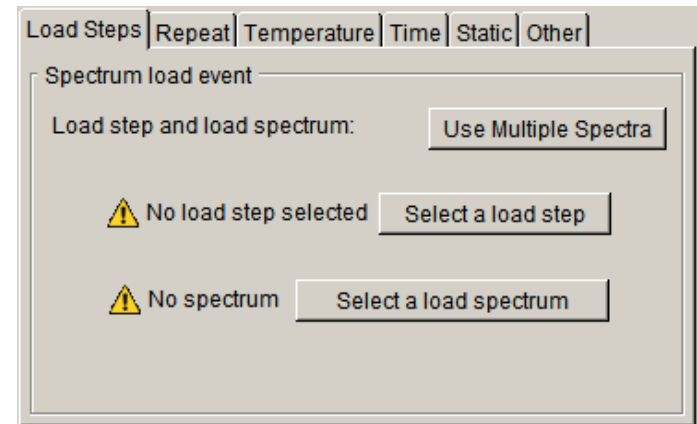
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$$(S_{\max}^{(n)}, S_{\min}^{(n)}) \rightarrow (K_{\max}^{(n)}, K_{\min}^{(n)})$$

Load Events

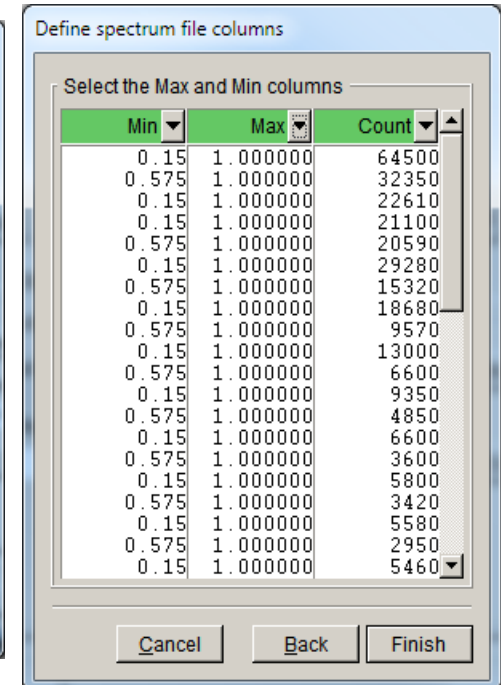
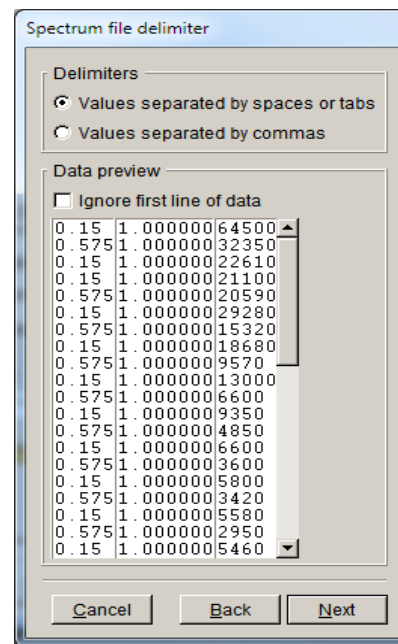
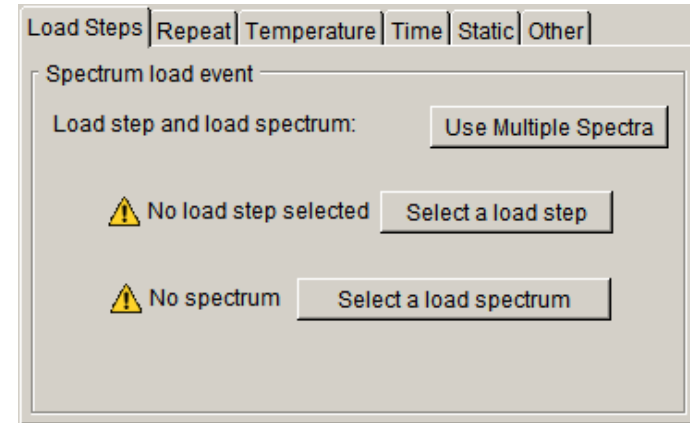
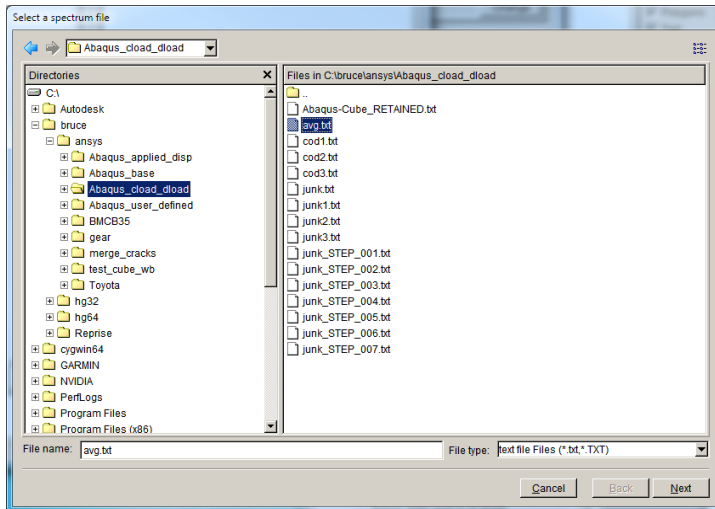
- **Spectrum load event:**
 - User must specify the load steps (and substeps if more than one) along with a load spectrum
 - Multiple spectra can be used
 - Multipliers and temperature offset can be applied



Load Events

Spectrum load event:

- Selecting load spectrum requires the spectrum data to be read from a file

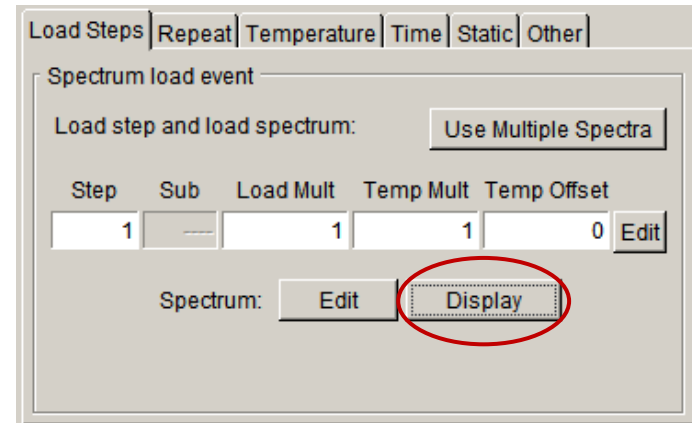


Data displayed in a table but can also be plotted in a graph.

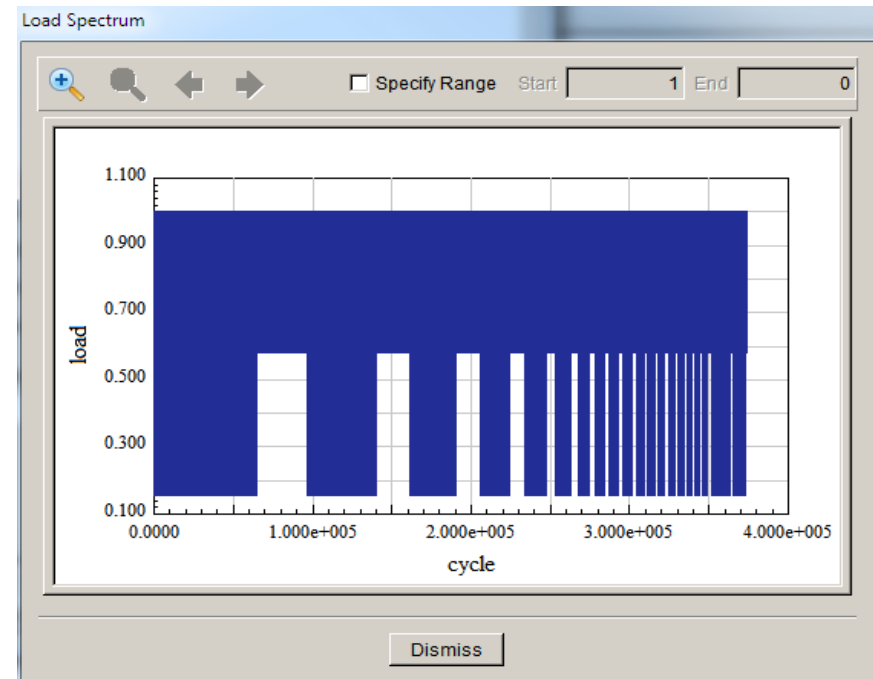


Load Events

- **Spectrum load event:**
 - Once the spectrum has been read, the Load schedule dialog appears as



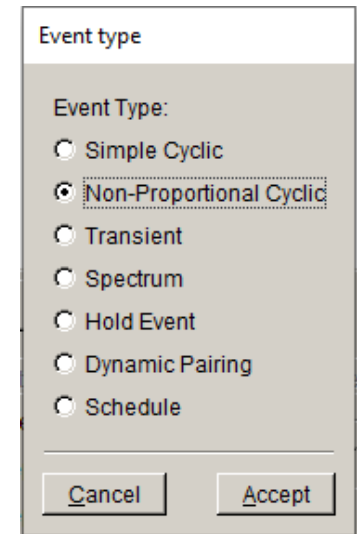
Display button



Load Events

- **Hold Event:**

- A hold event allows one to define a hold time without any companion load cycling
- Simple cyclic, Non-proportional cyclic, transient and spectrum loading events can include a hold (dwell) time
- If the Hold Event is chosen the user must specify the hold time and the load step

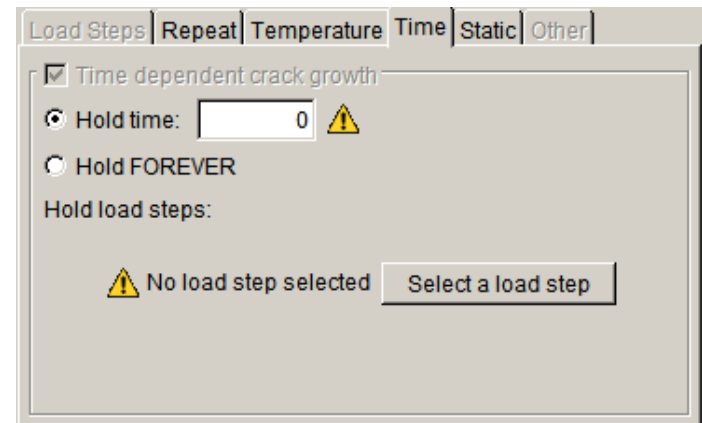


Event type

Event Type:


- Simple Cyclic
- Non-Proportional Cyclic
- Transient
- Spectrum
- Hold Event
- Dynamic Pairing
- Schedule

Cancel Accept




Load Steps Repeat Temperature Time Static Other

Time dependent crack growth

Hold time: 

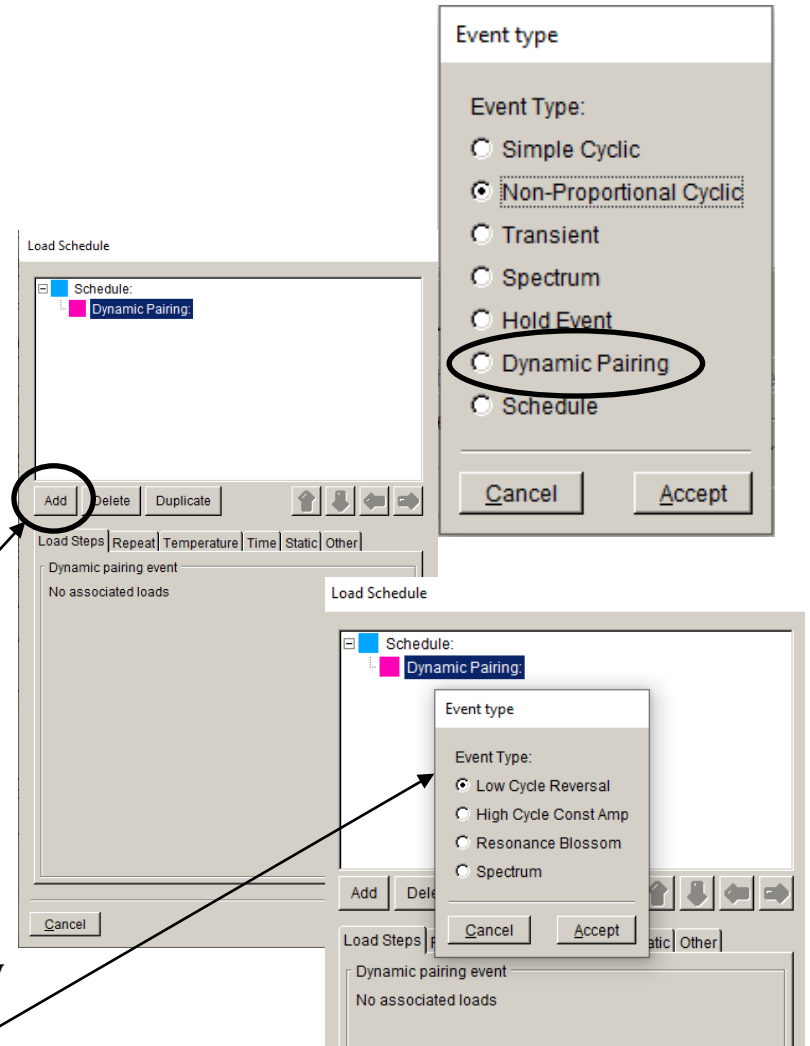
Hold FOREVER

Hold load steps:

 No load step selected

Load Events

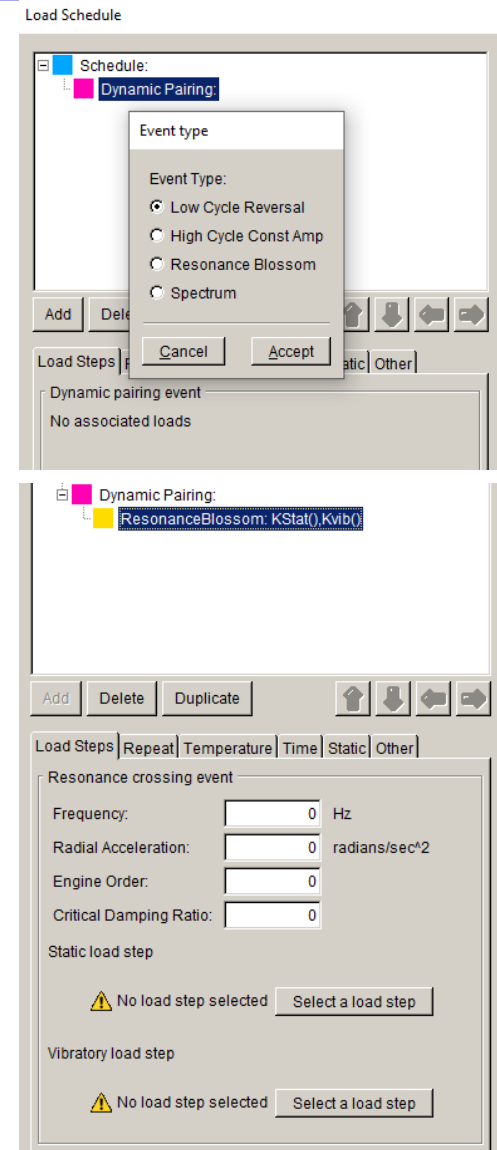
- **Dynamic Pairing:**
 - Examines all SIFs and dynamically creates max-min pairs
 - There are four event types:
 - Low Cycle Reversal – the load step associated with a LCF event is specified
 - High Cycle Const Amp – the load step associated with both a static and a vibratory load step are specified
 - Resonance Blossom
 - Spectrum



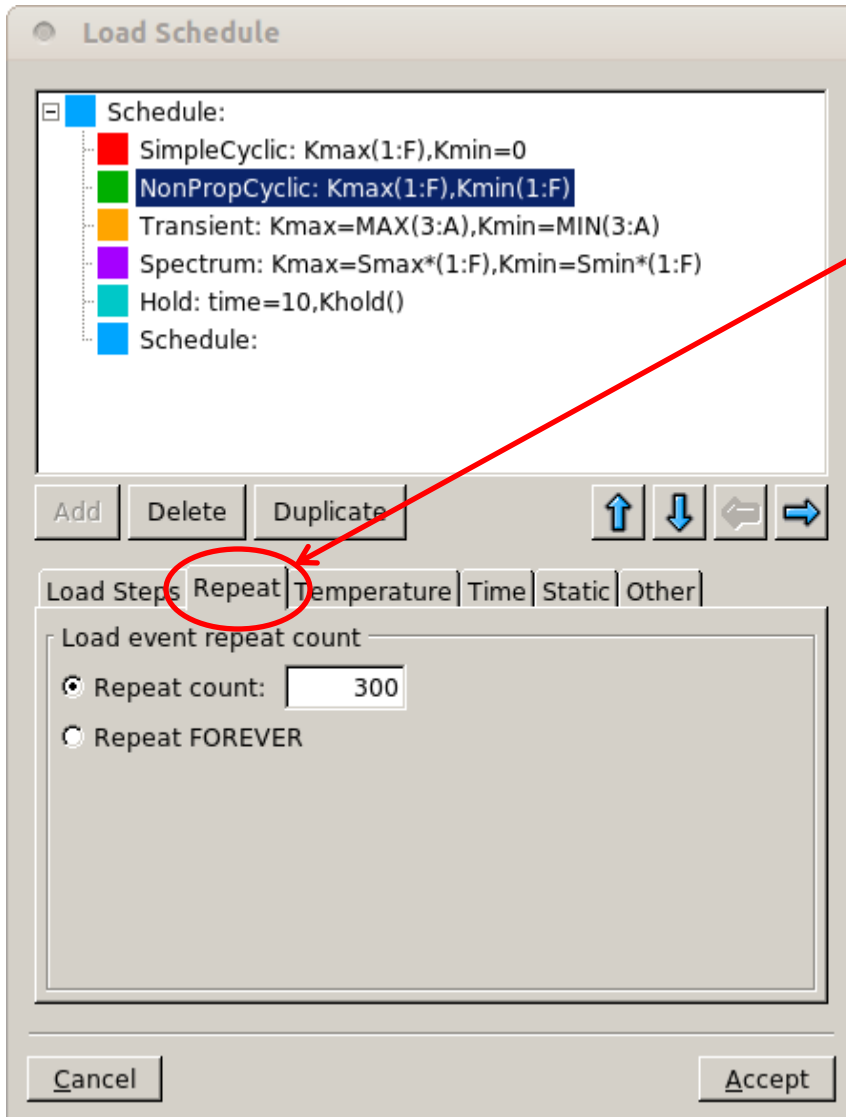
Use the **Add** button to display the Dynamic Pairing events.

Load Events

- **Dynamic Pairing:**
 - Resonance Blossom – the load step associated with a static load step and a vibratory load step are specified along with several parameters

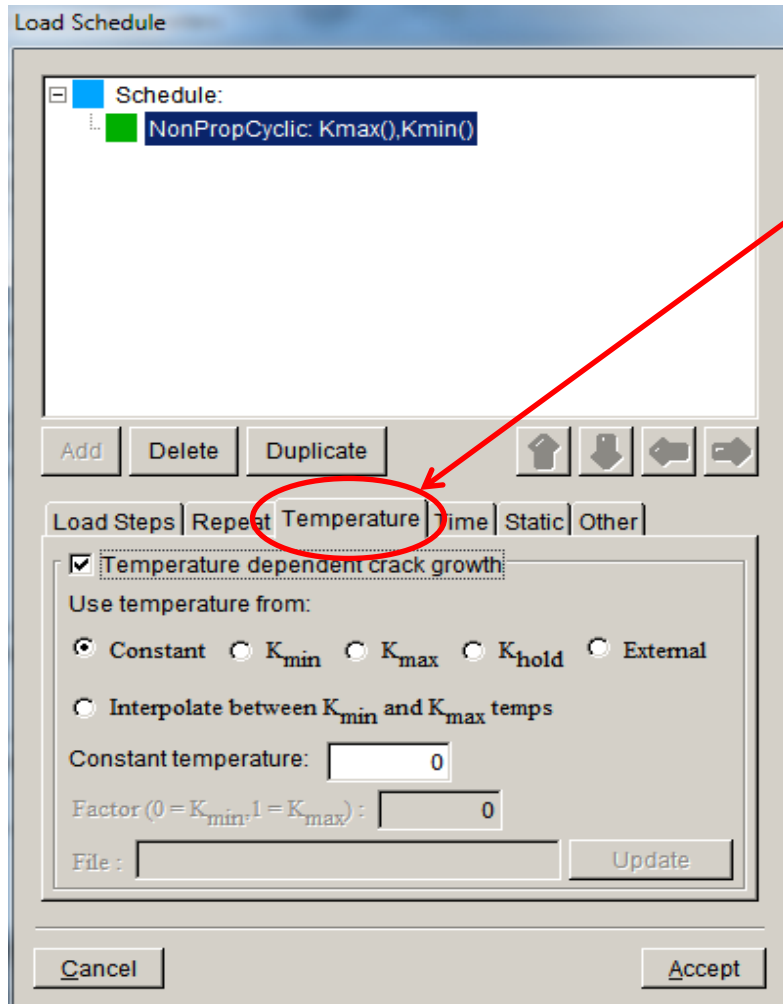


Repeat Tab



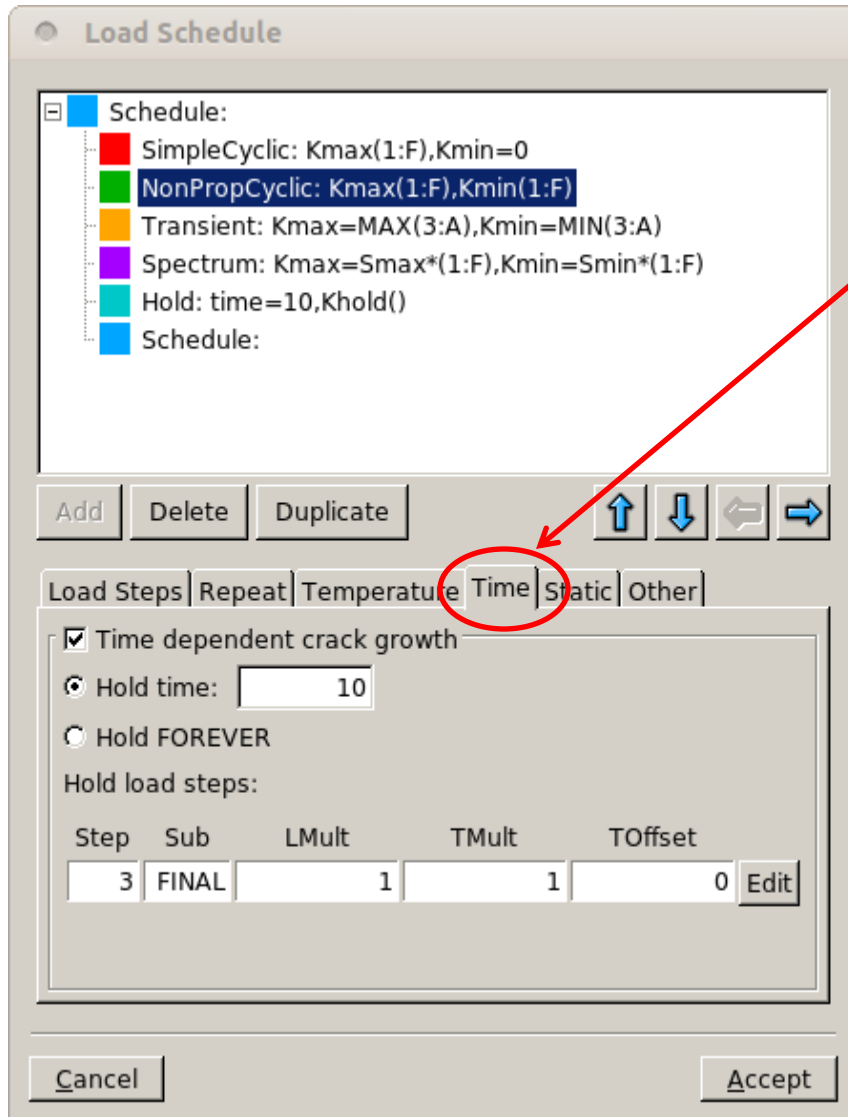
- Load events can be applied for a set number of times, or can be repeated (FOREVER) until K_{\max} equals K_{critical}

Temperature Tab



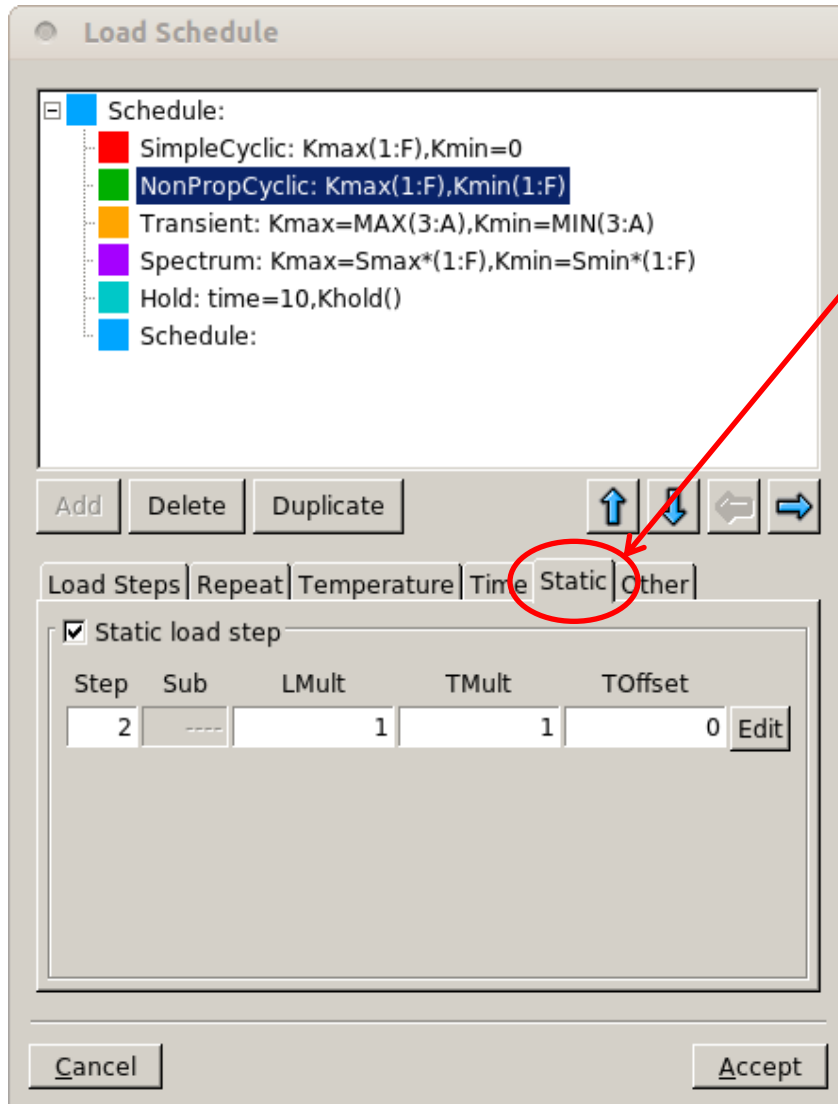
- For temperature dependent growth rate models, the temperature can be:
- Constant
 - Temperature at K_{\max}
 - Temperature at K_{\min}
 - Temperature at K_{hold}
 - External – crack-front temperatures are read from an independent set of analysis results
 - Interpolated between K_{\min} and K_{\max}

Time Tab



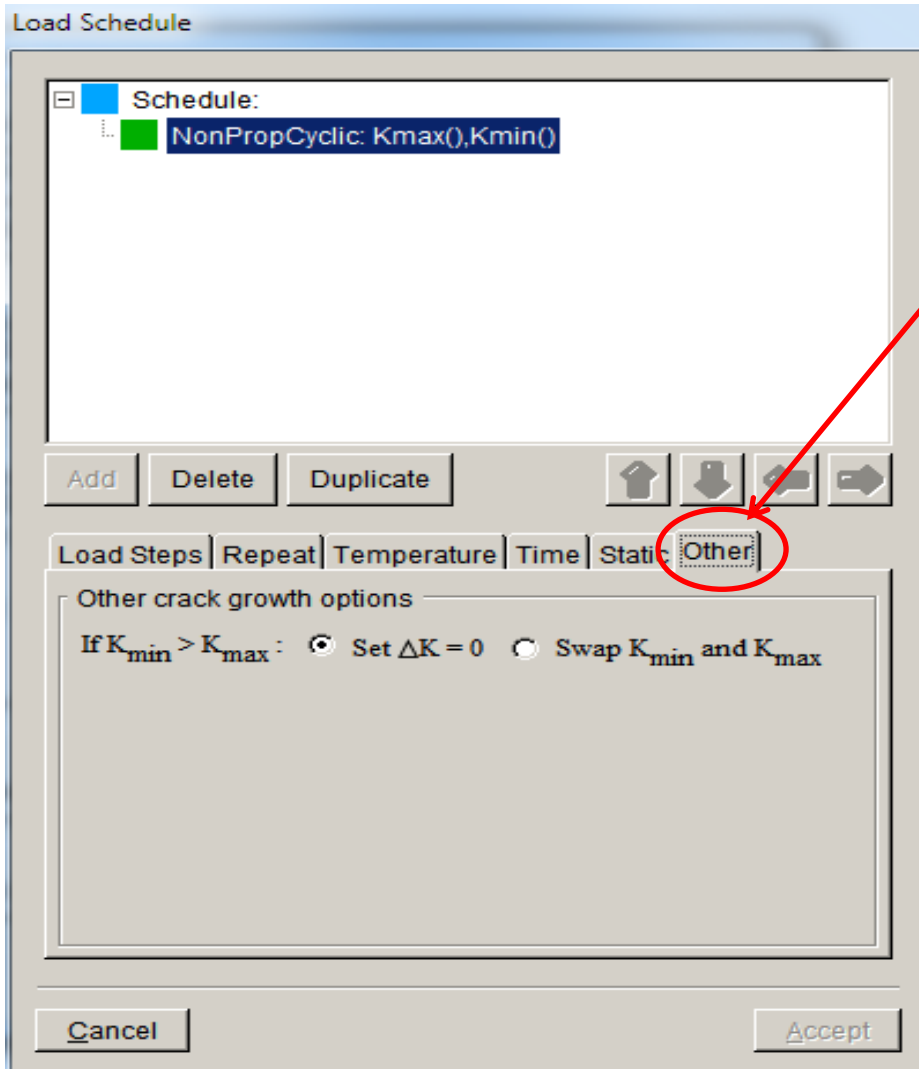
- For hold (dwell) times, K_{hold} can be computed from a single FE load step (sub-step), or from a combination of steps.
- Hold times can be for a fixed duration or until K_{hold} equals $K_{critical}$ (e.g. for stress corrosion cracking).

Static Tab



- K_{static} is added to both K_{max} and K_{min} to model residual stresses or other types of non-cyclic loads.

Other Tab

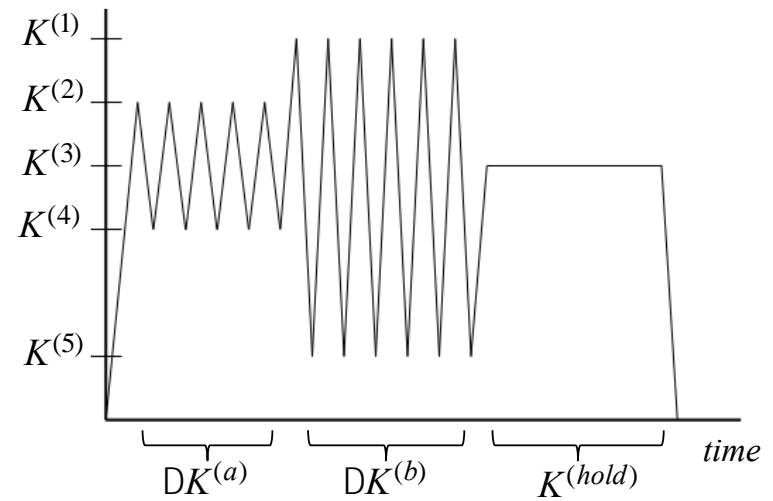


- Define what happens if $K_{min} > K_{max}$, which might happen for a transient event (for example)

Example: Simple Cyclic and Hold Load Events

- This load schedule consists of three load events
 - Two events are cyclic load events and they correspond to the stress intensity factor ranges $\Delta K^{(a)}$ and $\Delta K^{(b)}$
 - Hold event, where the load is held constant for a fixed length of time
- Linear finite element analysis was performed with one load step (load step 1 corresponds to $K^{(1)}$)
- Other stress intensity factors can be found as

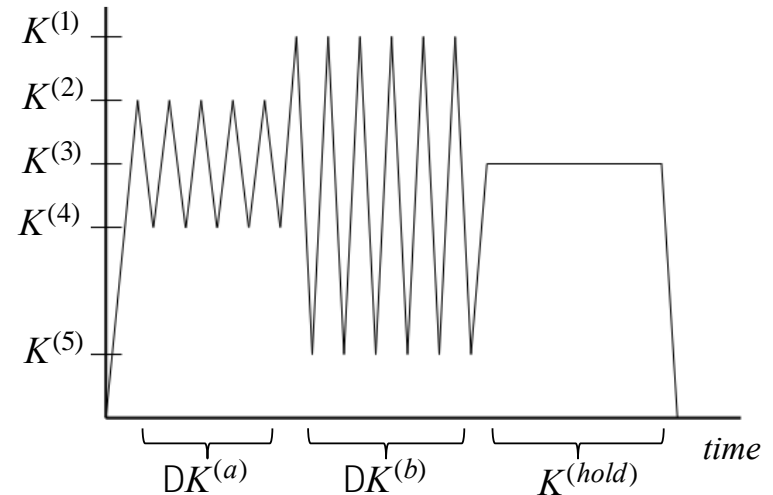
$$K^{(2)} = \frac{5K^{(1)}}{6}, \quad K^{(3)} = \frac{2K^{(1)}}{3}, \quad K^{(4)} = \frac{K^{(1)}}{2}, \quad \text{and} \quad K^{(5)} = \frac{K^{(1)}}{6}$$



stress intensity factor time history for a simple load schedule

Example: Simple Cyclic and Hold Load Events

- For a proportional cyclic load event, the minimum amount of information that must be specified to define an event is
 - Load step number associated with the maximum stress intensity factor, K_{\max} ,
 - Scaling multiplier,
 - Stress ratio, R ,
 - Repeat count.



stress intensity factor time history
for a simple load schedule

Event	Load Step	Multiplier	$R = K_{\min} / K_{\max}$	Repeat Count	Hold Time
1	1	5/6	3/5	5	N/A
2	1	1	1/6	6	N/A
3	1	2/3	N/A	N/A	t

Crack Growth Rate Models For Subcritical Crack Growth

Crack Growth Rate Model

- Subcritical Growth Parameters dialog requires the definition of Crack Growth Rate Model
- Select **New Model** button

Subcritical Growth Parameters

Units used in FE model
stress: MPa length: mm temp: C time: sec Change

Crack Growth Load Schedule
New Schedule Read From File Wizard View/Edit Save To File

Crack Growth Rate Model
New Model Read From File View/Edit da/dN View/Edit da/dt Save To File

Mixed-mode equivalent K
 $K^{\text{equiv}} = K_I$ $K^{\text{equiv}} = \sqrt{K_I^2 + (\gamma_{II} K_{II})^2 + (\gamma_{III} K_{III})^2}$
 $K^{\text{equiv}} = K_{\text{RSS}}$ γ_{II} : γ_{III} :
sign: from K_I from K_{II} from K_{III} always positive always negative

Effective Delta K
 $\Delta K_{\text{eff}} = K_{\text{max}} - K_{\text{min}}$ $\Delta K_{\text{eff}} = K_{\text{max}} - \max(K_{\text{min}}, 0)$
 Use load interaction model Set/Edit Parameters

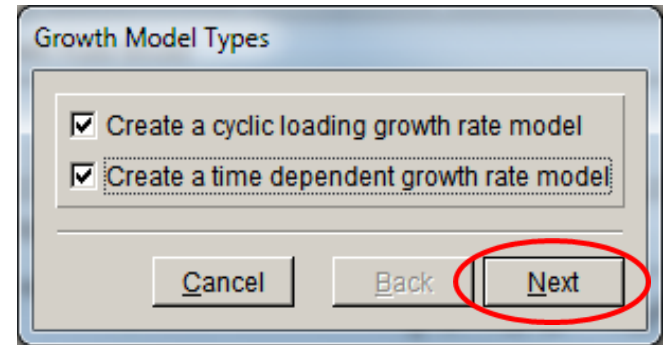
Integration Options
 Accelerated counting Constant K for time integration

Dynamic Pairing Metric
 $\Delta K_{\text{eff}}^{\text{equiv}}$ da/dN ΔCTD

Cancel Back Finish

Crack Growth Rate Model

- Cyclic loading and time dependent growth rate models can be defined
- User can select cyclic loading or time dependent or both
- Click on the **Next** button to select the crack growth model



Cyclic Loading Growth Model

- Select the cyclic loading growth model and the R-ratio function
- For the Paris crack growth rate model, for example, with **no stress ratio effects**, the next dialog allows you specify either a temperature dependent or independent model
 - For a temperature independent model, only one set of growth rate parameters (C and m for the Paris model) are required
 - For a temperature dependent model, growth model parameters are specified for each temperature

Cyclic loading growth model

Cyclic loading growth model

Paired growth rate and R-ratio functions

Growth rate model: Paris
 Bilinear Paris
 Sigmoidal
 Hyperbolic sine
 Table lookup

Stress ratio model: None
 Walker equation
 Newman closure
 Table lookup

NASGRO version 4 equation

Key in
 NASGRO user XML mat file

NASGRO version 3 equation
 Modified Hartman-Schijve model
 User defined model

Cyclic Loading Growth Model

Temperature independent model
 Temperature dependent model

Paris Growth Rate Model

- Paris growth rate model $da/dN = C(\Delta K_{eff})^n$

The parameters C and n must be specified along with values for $\Delta K_{threshold}$ and $K_{critical}$.

If $\Delta K_{eff} \leq \Delta K_{threshold}$, da/dN is set to zero

If $\Delta K_{eff} \geq (1 - R)K_{critical}$, unstable crack growth is assumed and da/dN is infinite

For temperature independent model

Growth Rate Model

Paris growth model: Plot

$$da / dN = C \Delta K^n$$

Model Label (Optional):

Description (Optional):

Units: stress: MPa length: mm temp: C time: min Change

	C	n	DKth	Kc
1	4e-11	3	1	1e+004

Cancel Accept

Units must be specified; if the units are different than FE Model units, **conversions are done automatically**

For temperature dependent model

Cyclic Loading Growth Model

Paris growth model: Plot

$$da / dN = C \Delta K^n$$

Model Label (Optional):

Description (Optional):

Units: stress: unset length: unset temp: unset time: unset Change

Number of temperatures: File Interpolate Closest Next highest

Temperature	C	n	DKth	Kc
1				
2				
3				
4				
5				

Cancel Back Next

Options are available for interpolating between temperatures

Paris Growth Rate Model

- Paris growth rate model with Walker Stress Ratio model

The parameters $m+$ and $m-$ must be specified

$$da/dN = C(\Delta K_{eff})^n$$

$$\Delta K_{eff} = (1 - R)^{m-1} \Delta K$$

$$m = \begin{cases} m+ & R \geq 0 \\ m- & R < 0 \end{cases}$$

For temperature independent model with Walker stress ratio model

For temperature dependent model with Walker stress ratio model

Cyclic Loading Growth Model

Paris growth model: $da/dN = C(\Delta K_{eff})^n$

Walker stress ratio model: $\Delta K_{eff} = \Delta K(1-R)^{m^+-1}$ for $R > 0$
 $\Delta K_{eff} = \Delta K(1-R)^{m^- -1}$ for $R < 0$

Model Label (Optional):

Description (Optional):

Units: stress: unset length: unset temp: unset time: unset

	C	n	DKth	Kc	m_pos	m_neg
1	1					

Cyclic Loading Growth Model

Paris growth model: $da/dN = C(\Delta K_{eff})^n$

Walker stress ratio model: $\Delta K_{eff} = \Delta K(1-R)^{m^+-1}$ for $R > 0$
 $\Delta K_{eff} = \Delta K(1-R)^{m^- -1}$ for $R < 0$

Model Label (Optional):

Description (Optional):

Units: stress: unset length: unset temp: unset time: unset

Number of temperatures: 5

Temperature	C	n	DKth	Kc	m_pos	m_neg
1						
2						
3						
4						
5						

Units must be specified

Paris Growth Rate Model

- Paris growth rate model with Table lookup stress ratio model

$$R_1, (C, n, \Delta K_{threshold}, K_{critical})_1$$

$$R_2, (C, n, \Delta K_{threshold}, K_{critical})_2$$

$$R_3, (C, n, \Delta K_{threshold}, K_{critical})_3$$

⋮

$$da/dN = C(\Delta K_{eff})^n$$

$$\Delta K_{eff} \geq (1 - R)K_{critical}$$

For temperature independent model with Table-lookup stress ratio model

Units must be specified

Cyclic Loading Growth Model

Paris growth model: $da / dN = C (\Delta K_{eff})^n$

Stress ratio model: Table lookup for growth parameters Plot

Model Label (Optional):

Description (Optional):

Units: stress: unset length: unset temp: unset time: unset ⚠ Change

Number of R values: File

	R	C	n	DKth	Kc
1					
2					
3					
4					
5					

Cancel Back Next

Paris Growth Rate Model

- Paris growth rate model with **Table lookup stress ratio model**

$$R_1, (C, n, \Delta K_{threshold}, K_{critical})_1$$

$$R_2, (C, n, \Delta K_{threshold}, K_{critical})_2$$

$$R_3, (C, n, \Delta K_{threshold}, K_{critical})_3$$

$$\vdots$$

$$da/dN = C(\Delta K_{eff})^n$$

$$\Delta K_{eff} \geq (1 - R)K_{critical}$$

For temperature dependent model with Table-lookup stress ratio model

Cyclic Loading Growth Model

Paris growth model: $da/dN = C(\Delta K_{eff})^n$ Stress ratio model: Table lookup for growth parameters

Model Label (Optional):

Description (Optional):

Units: stress: unset length: unset temp: unset time: unset ⚠ Change

Number of temperatures: Interpolate Closest Next highest

	Temperature	data
1	70	table data
2	500	table data
3		table data
4		table data
5		table data

Cancel Back Next

Enter/Edit data

Number of R values: File

	R	C	n	DKth	Kc
1					
2					
3					
4					
5					

Cancel Accept

Enter/Edit data

Number of R values: File

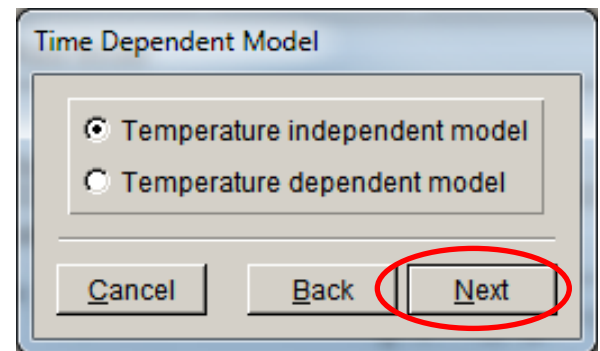
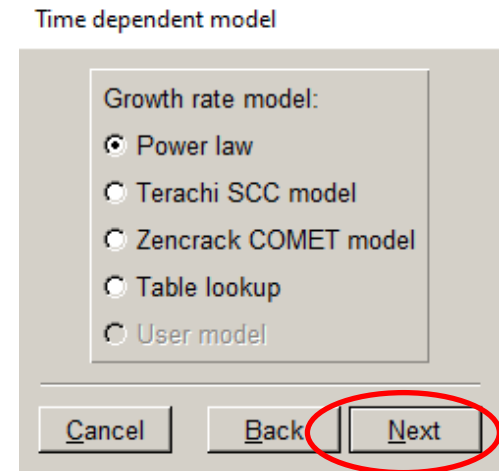
	R	C	n	DKth	Kc
1					
2					
3					
4					
5					

Cancel Accept

Units must be specified

Time Dependent Growth Model

- FRANC3D supports the following time-dependent growth models:
 - Power Law
 - Terachi SCC
 - Zencrack COMET
 - Table Lookup
- Some of these growth models can be temperature dependent or independent.



Power Law Growth Model

- Power law growth model $da/dt = AK^m$
- For time dependent rates, the threshold (K_{th}) is an actual K value rather than a ΔK value

Temperature independent, time-dependent power law data

Growth Rate Model

Growth rate model: $da / dt = A K^m$ Plot

Model Label (Optional):

Description (Optional):

Units: stress: MPa length: mm temp: C time: min Change

	A	m	Kth	Kc
1	5e-018	5	1	1e+004

Cancel Accept

Units must be specified

Temperature dependent, time-dependent Power law data

Time Dependent Growth Model

Growth rate model: $da / dt = A K^m$ Plot

Model Label (Optional):

Description (Optional):

Units: stress: unset length: unset temp: unset time: unset Change

Number of temperatures: File Interpolate Closest Next highest

	Temperature	A	m	kth	Kc
1					
2					
3					
4					
5					

Cancel Back Finish

Mixed-mode equivalent K

- The equivalent K or ΔK can be defined using only Mode I (K_I) or using all three Modes
- The sign for K^{equiv} can be set based on the sign of K_I , K_{II} or K_{III} or can be set as always positive or always negative.

Subcritical Growth Parameters

Units used in FE model
stress: unset length: unset temp: unset time: unset Change

Crack Growth Load Schedule
New Schedule Read From File Wizard View/Edit Save To File

Crack Growth Rate Model
New Model Read From File View/Edit da/dN View/Edit da/dt Save To File

Mixed-mode equivalent K

$K^{\text{equiv}} = K_I$ $K^{\text{equiv}} = \sqrt{K_I^2 + (\gamma_{II} K_{II})^2 + (\gamma_{III} K_{III})^2}$

γ_{II} : γ_{III} :

sign: from K_I from K_{II} always positive always negative

Effective Delta K

$\Delta K_{\text{eff}} = K_{\text{max}} - K_{\text{min}}$ $\Delta K_{\text{eff}} = K_{\text{max}} - \max(K_{\text{min}}, 0)$

Integration Options

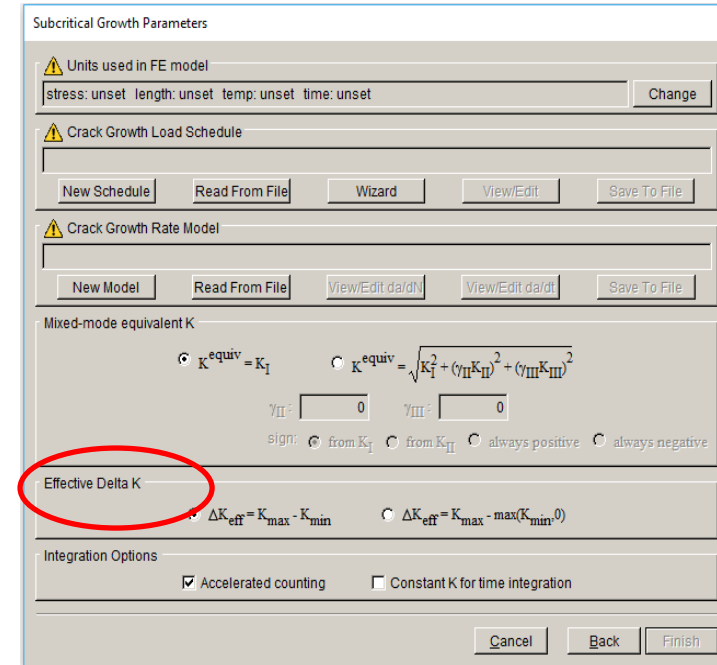
Accelerated counting Constant K for time integration

Cancel Back Finish

Effective ΔK

- ΔK Computations:

- Normally it is computed as $\Delta K = K_{\max} - K_{\min}$
- If the minimum loading at the crack front is compressive and contact elements are not used at the crack faces to prevent crack overlap
 - computed K_{\min} and corresponding stress ratio (R) will be negative
 - crack face overlap is physically unrealistic
 - negative values can be accepted if the appropriate parameters are used to predict the crack growth



Effective ΔK

- FRANC3D can compute a more physically realistic ΔK that only considers portions of a load cycle where the K 's are positive

$$\Delta K = \begin{cases} K_{\max} - K_{\min} & \text{if } K_{\min} > 0 \\ K_{\max} & \text{if } K_{\min} \leq 0 \end{cases}$$

- One option is for FRANC3D to check for a closed crack for the complete cycle and to set the stress intensity factor range and R to zero:

$$\Delta K_{\text{effective}}^{\text{equivalent}} = \begin{cases} 0 & K_{\max}^{\text{equivalent}} \leq 0 \\ \Delta K^{\text{equivalent}} & K_{\max}^{\text{equivalent}} > 0 \end{cases}$$

$$R_{\text{effective}}^{\text{equivalent}} = \begin{cases} 0 & K_{\max}^{\text{equivalent}} \leq 0 \\ R^{\text{equivalent}} & K_{\max}^{\text{equivalent}} > 0 \end{cases}$$

Effective ΔK

- FRANC3D allows the option to truncate the stress intensity factor range to eliminate the negative (closed) portion of the cycle. In this case, the expressions are:

$$\Delta K_{effective}^{equivalent} = \begin{cases} 0 & K_{max}^{equivalent} \leq 0 \\ \Delta K^{equivalent} & K_{min}^{equivalent} \geq 0 \\ K_{max}^{equivalent} & K_{min}^{equivalent} < 0 \end{cases}$$

$$R_{effective}^{equivalent} = \begin{cases} 0 & K_{max}^{equivalent} \leq 0 \\ R^{equivalent} & K_{min}^{equivalent} \geq 0 \\ 0 & K_{min}^{equivalent} < 0 \end{cases}$$

Integration Options

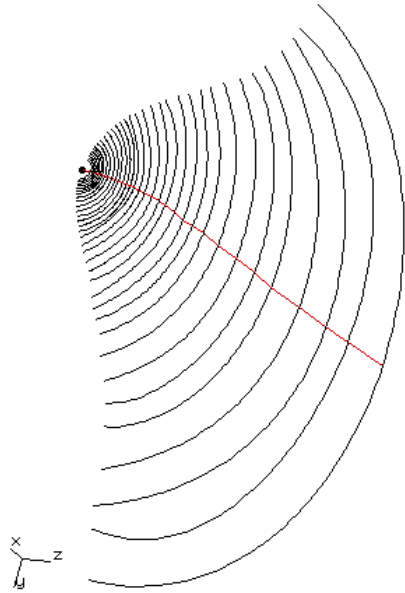
- Crack growth integrations are performed using either:
 - Accelerated counting (for cycling loading)
 - If it is selected (the default), FRANC3D will try to use an accelerated cycle counting algorithm based on Runge-Kutta integration (might be slightly less accurate but is much faster)
 - If it is not selected, FRANC3D will perform cycle-by-cycle counting (more accurate but can be time consuming)
 - Constant K for time integration
 - SIF will be function of time

$$\Delta a = \int_0^t A \cdot K_{hold}(\tau)^m d\tau$$
 - If the crack growth is very small over the hold time, the above equation can be simplified to

$$\Delta a = A \cdot K_{hold}^m \cdot t$$

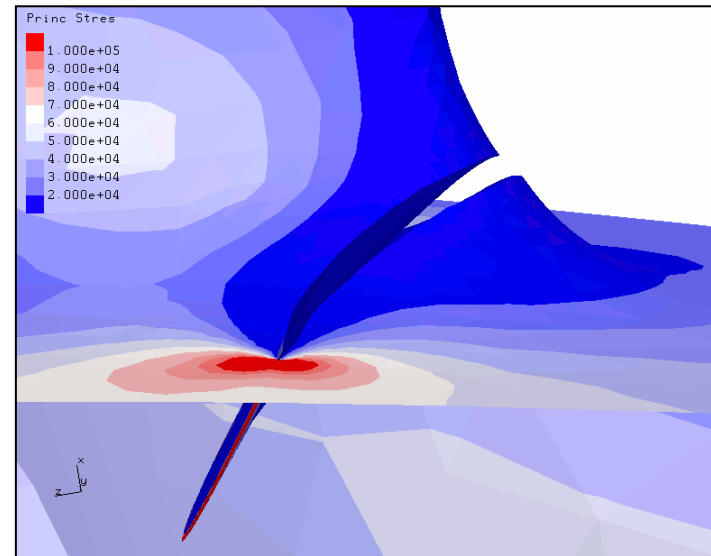
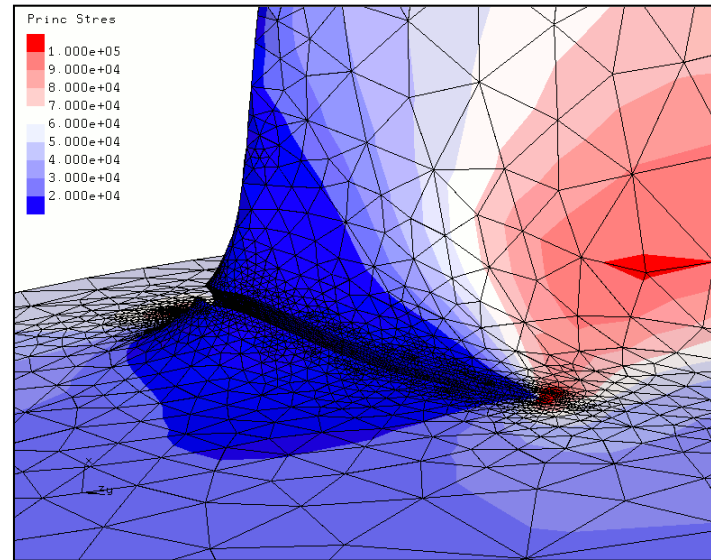
The screenshot shows the 'Subcritical Growth Parameters' dialog box in FRANC3D. The 'Integration Options' section is circled in red. It contains two checkboxes: 'Accelerated counting' (checked) and 'Constant K for time integration' (unchecked). Other sections include 'Units used in FE model' (stress: MPa, length: mm, temp: C, time: sec), 'Crack Growth Load Schedule', 'Crack Growth Rate Model', 'Mixed-mode equivalent K' (with radio buttons for \$K_I\$, \$K_{RSS}\$, and a square root formula, and sign options), 'Effective Delta K' (with radio buttons for \$\Delta K_{eff} = K_{max} - K_{min}\$ and \$\Delta K_{eff} = K_{max} - \max(K_{min}, 0)\$), and 'Dynamic Pairing Metric' (with radio buttons for \$\Delta K_{eff}^{equiv}\$, \$da/dN\$, and \$\Delta CIOD\$).

Simulation of Crack Growth



predicted crack fronts (steps)

Crack steps are a sequence of finite element models used to simulate crack growth



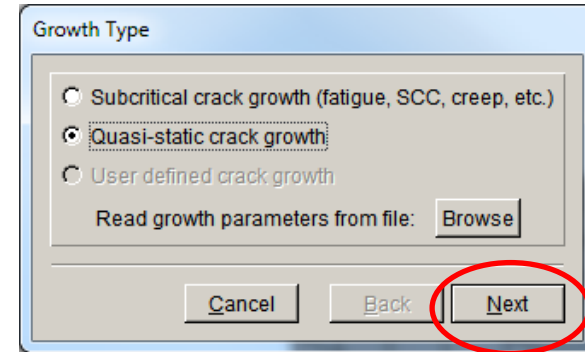
Quasi-Static Crack Growth

Quasi-static Crack Growth

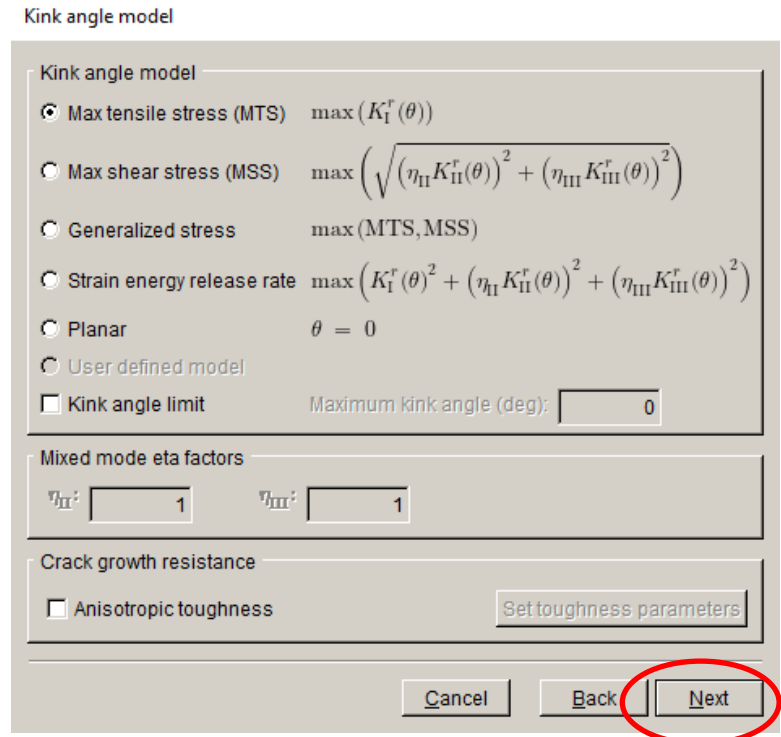
- Stress intensity factor of one or more of the points on the crack front is equal to a critical value
- It is assumed that stress intensity factors can never be greater than a critical value
- Used for situations where the rate of change of the stress intensity factors with respect to crack growth is negative, and we need to increase the applied load to keep a crack growing
- Quasi-static growth can be used if fatigue data/model is not available

FRANC3D Quasi-Static Crack Growth Procedure

- Select Quasi-static crack growth
- Click **Next**



- Select kink angle model
- Click **Next**



FRANC3D Quasi-Static Crack Growth Procedure

- Specifying quasi-static growth parameters:
 - Power (n) for the growth model
 - K^{equiv}
 - Load step(s)
- Click **Finish**

Quasi-static Growth Parameters

Power Law Growth Parameter

$$\Delta a_i = \Delta a_{\text{median}} (K_i / K_{\text{median}})^n$$

n:

Mixed-mode equivalent K

$K^{\text{equiv}} = K_I$ $K^{\text{equiv}} = \sqrt{K_I^2 + (\gamma_{II} K_{II})^2 + (\gamma_{III} K_{III})^2}$

γ_{II} : γ_{III} :

sign: from K_I from K_{II} always positive always negative

FEM Load Steps

No load step selected

Quasi-static Growth Parameters

Power Law Growth Parameter

$$\Delta a_i = \Delta a_{\text{median}} (K_i / K_{\text{median}})^n$$

n:

Mixed-mode equivalent K

$K^{\text{equiv}} = K_I$ $K^{\text{equiv}} = \sqrt{K_I^2 + (\gamma_{II} K_{II})^2 + (\gamma_{III} K_{III})^2}$

γ_{II} : γ_{III} :

sign: from K_I from K_{II} always positive always negative

FEM Load Steps

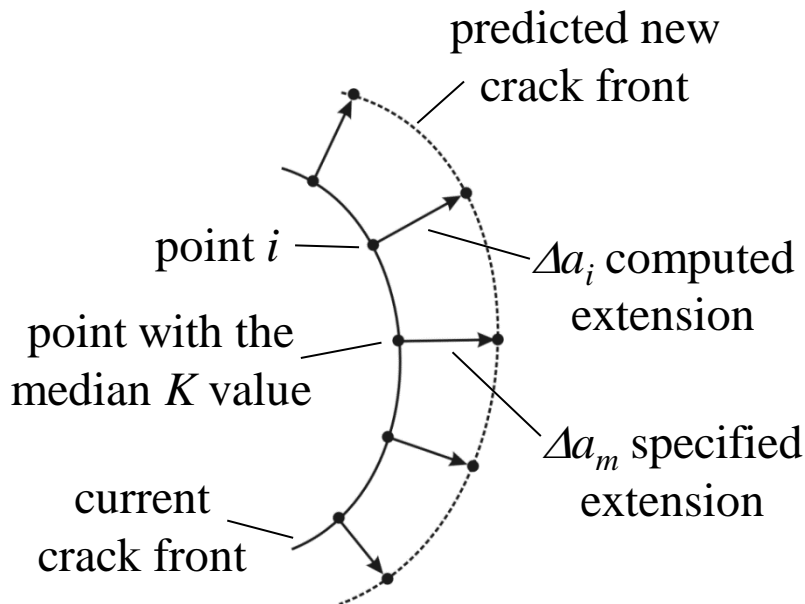
Step	Sub	Load Mult	Temp Mult	Temp Offset	
1	---	1	1	0	<input type="button" value="Edit"/>

FRANC3D Quasi-Static Crack Growth Model

After the crack growth model has been defined, FRANC3D will display the computed crack growth:

- Crack extension can be adjusted if needed
- A scale node of 0.5 corresponds to the median

$$\Delta a_i = \Delta a_m \left(\frac{\Delta K_i}{\Delta K_m} \right)^n$$

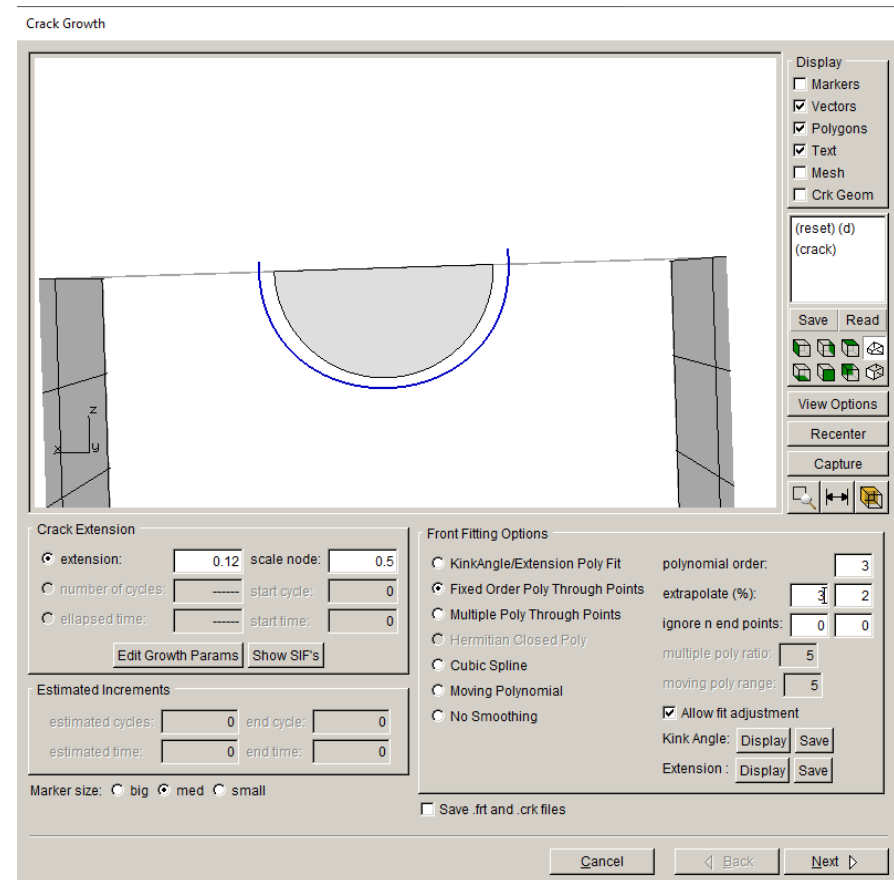


The screenshot shows the 'Crack Growth' software interface. The main window displays a 3D model of a crack in a rectangular block. The crack is shown as a semi-circular front. The software interface includes several control panels:

- Crack Extension:** Contains fields for 'extension' (0.12), 'scale node' (0.5), 'number of cycles', 'elapsed time', 'start cycle', and 'start time'. The 'extension' and 'scale node' fields are circled in red.
- Front Fitting Options:** Includes radio buttons for 'KinkAngle/Extension Poly Fit', 'Fixed Order Poly Through Points', 'Multiple Poly Through Points', 'Hermitian Closed Poly', 'Cubic Spline', 'Moving Polynomial', and 'No Smoothing'. It also has a 'polynomial order' field (3) and 'extrapolate (%)' (2).
- Estimated Increments:** Fields for 'estimated cycles' and 'estimated time'.
- Display:** Checkboxes for 'Markers', 'Vectors', 'Polygons', 'Text', 'Mesh', and 'Crk Geom'.
- Buttons:** 'Save', 'Read', 'View Options', 'Recenter', 'Capture', 'Cancel', 'Back', and 'Next'.

FRANC3D Quasi-Static Crack Growth Model

- Computed new crack front points are smoothed and extrapolated; interior cracks do not require extrapolation
- Front fitting options include:
 - Polynomial and spline curve fits
 - Polynomial order (default is 3)
 - Fields for extrapolating the ends of the fitted curves, and the option to discard crack front points at either end
- Save .frt and .crk files option allows the user to save the new front points and the new crack geometry to a file before inserting it into the model
 - .frt file can be read using **Read Crack Growth** wizard
 - .crk file can be read using **Read Flaw From Files** option



User-defined Crack Growth

- FRANC3D allows Python functions to define crack growth:
 - User-defined kink angle
 - User-defined crack growth model
- User-defined crack growth is activated if the user has pre-defined Python extensions enabled via the **Advanced** menu

Crack Growth Stopping Criteria

- Automatic crack growth stops when the first of the following is satisfied: $N \geq N_{\max}$ $K_{\max} \geq K_c$ $\Delta K \leq \Delta K_{\text{threshold}}$ $\text{step} \geq \text{max step}$
- These criteria require that FRANC3D maintain a running count of the applied cycles.

Fatigue Crack Growth: A Brief Summary of Options to Grow a Crack in Fatigue

FRANC3D Fatigue Crack Growth Procedure

Subcritical Growth Parameters

Units used in FE model
 stress: MPa length: mm temp: C time: sec Change

Crack Growth Load Schedule

New Schedule Read From File Wizard View/Edit Save To File

Crack Growth Rate Model

New Model Read From File View/Edit da/dN View/Edit da/dt Save To File

Mixed-mode equivalent K

$K^{equiv} = K_I$
 $K^{equiv} = \sqrt{K_I^2 + (\gamma_{II} K_{II})^2 + (\gamma_{III} K_{III})^2}$
 $K^{equiv} = K_{RSS}$
 γ_{II} γ_{III}
 sign: from K_I
 from K_{II}
 from K_{III}
 always positive
 always negative

Effective Delta K

$\Delta K_{eff} = K_{max} - K_{min}$
 $\Delta K_{eff} = K_{max} - \max(K_{min}, 0)$
 Use load interaction model Set/Edit Parameters

Integration Options

Accelerated counting
 Constant K for time integration

Dynamic Pairing Metric

ΔK_{eff}^{equiv}
 da/dN
 $\Delta CIOD$

Cancel Back Finish

Load Schedule

... **Schedule**

Event type

Event Type:

Simple Cyclic
 Non-Proportional Cyclic
 Transient
 Spectrum
 Hold Event
 Schedule

Add Delete

Load Steps Repeat

Load event repeat
 Repeat count
 Repeat FOREVER

Cancel Accept

Select Load Step

Load Step:

Load Sub-Step:

Load Multiplier:

Temperature Multiplier:

Temperature Offset:

Cancel Sum Multiple Steps Accept

Load Schedule

Add Delete Duplicate ↑ ↓ ← →

Load Steps Repeat Temperature Time Static Other

Nonproportional cyclic load event

K max load step:

⚠ No load step selected Select a load step

K min load step:

⚠ No load step selected Select a load step

Cancel Accept

FRANC3D Fatigue Crack Growth Procedure

Subcritical Growth Parameters

Units used in FE model
stress: MPa length: mm temp: C time: sec Change

Crack Growth Load Schedule

New Schedule Read From File Wizard View/Edit Save To File

Crack Growth Rate Model

New Model Read From File View/Edit da/dN View/Edit da/dt Save To File

Mixed-mode equivalent K

$K^{equiv} = K_I$ $K^{equiv} = \sqrt{K_I^2 + (\gamma_{II} K_{II})^2 + (\gamma_{III} K_{III})^2}$

$K^{equiv} = K_{RSS}$ γ_{II} : γ_{III} :

sign: from K_I from K_{II} from K_{III} always positive always negative

Effective Delta K

$\Delta K_{eff} = K_{max} - K_{min}$ $\Delta K_{eff} = K_{max} - \max(K_{min}, 0)$

Use load interaction model Set/Edit Parameters

Integration Options

Accelerated counting Constant K for time integration

Dynamic Pairing Metric

ΔK_{eff}^{equiv} da/dN $ACI008$

Cyclic Loading Growth Model

Pared growth rate and R-ratio functions

Growth rate model: Paris Bilinear Paris Sigmoidal Hyperbolic Sine Table Lookup

Stress ratio model: None Walker Equation Newman Closure Table Lookup

NASGRO version 4 equation

Key in NASGRO user XML mat file Select Browse

NASGRO version 3 equation

User defined model

Cancel Back Next

Cyclic Loading Growth Model

Temperature independent model Temperature dependent model

Cancel Back Next

Growth Rate Model

Paris growth model: Plot

$$da / dN = C \Delta K^n$$

Model Label (Optional):

Description (Optional):

Units: stress: MPa length: mm temp: C time: min Change

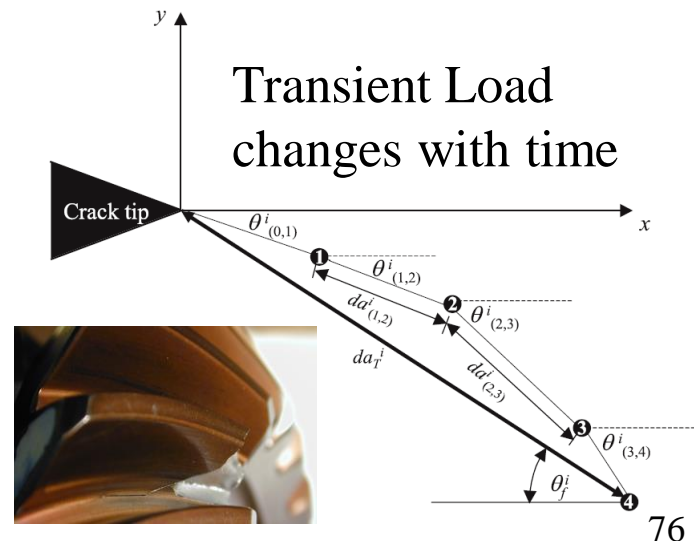
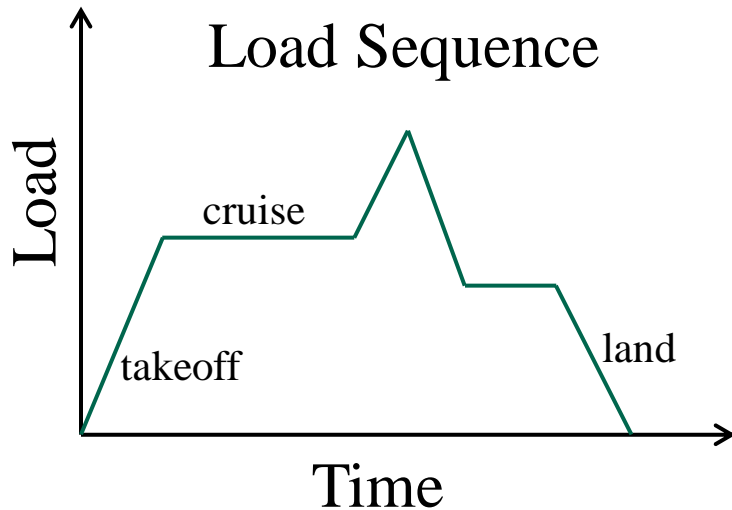
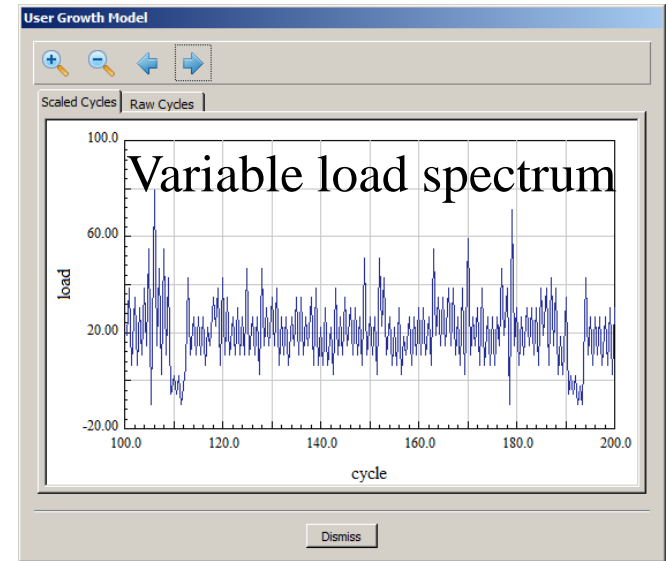
	C	n	DKth	Kc
1	4e-11	3	1	1e+004

Cancel Accept

Variable Amplitude Loads

Variable amplitude:

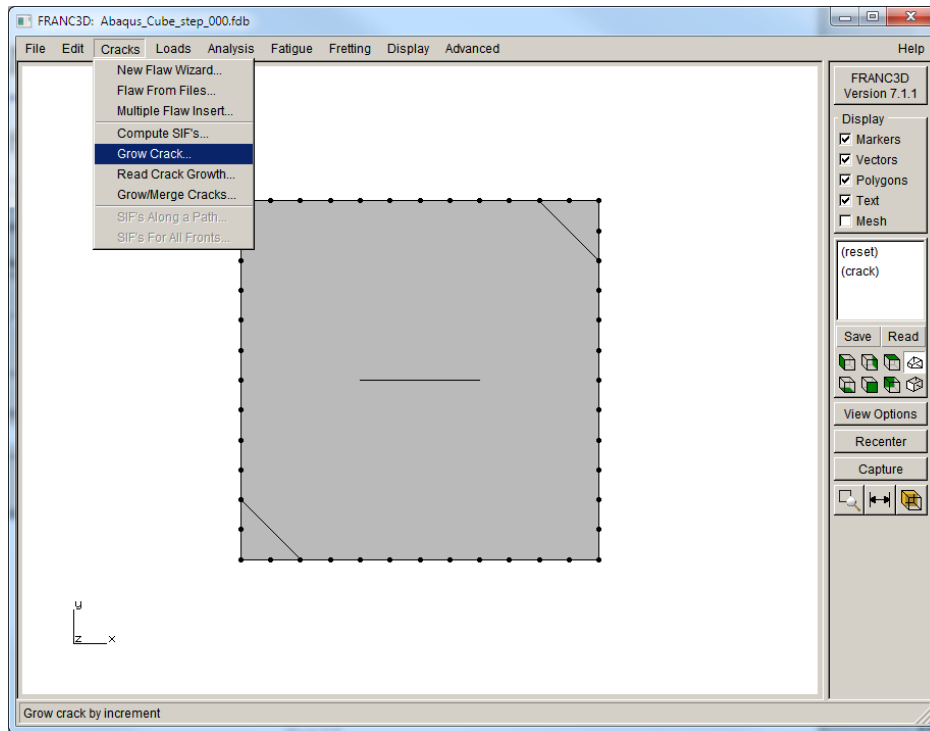
- Spectrum
- Sequence
- Transient



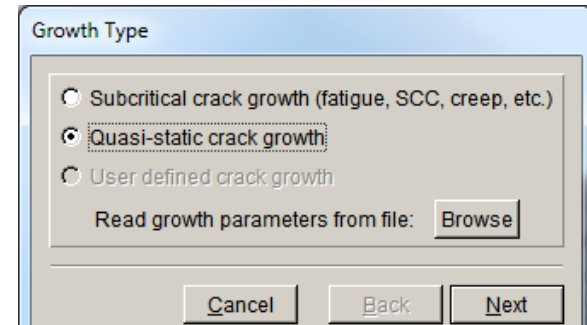
Demo/Hands On (Homework): Crack Growth

FRANC3D Tutorial 1 – Manual Crack Growth

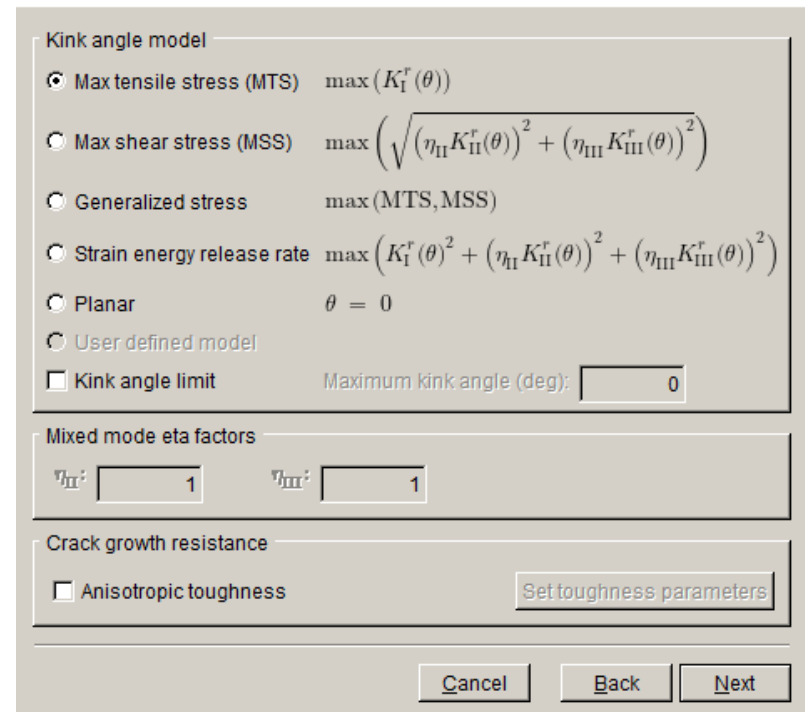
From FRANC3D menu, select
Cracks - Grow Crack



Choose quasi-static crack growth
and use MTS kink angle rule.



Kink angle model



FRANC3D Tutorial 1 – Manual Crack Growth

Specify Growth Rate parameters

Quasi-static Growth Parameters

Power Law Growth Parameter

$$\Delta a_i = \Delta a_{\text{median}} \left(\frac{K_i}{K_{\text{median}}} \right)^n$$

n:

Mixed-mode equivalent K

$K^{\text{equiv}} = K_I$ $K^{\text{equiv}} = \sqrt{K_I^2 + (\gamma_{II} K_{II})^2 + (\gamma_{III} K_{III})^2}$

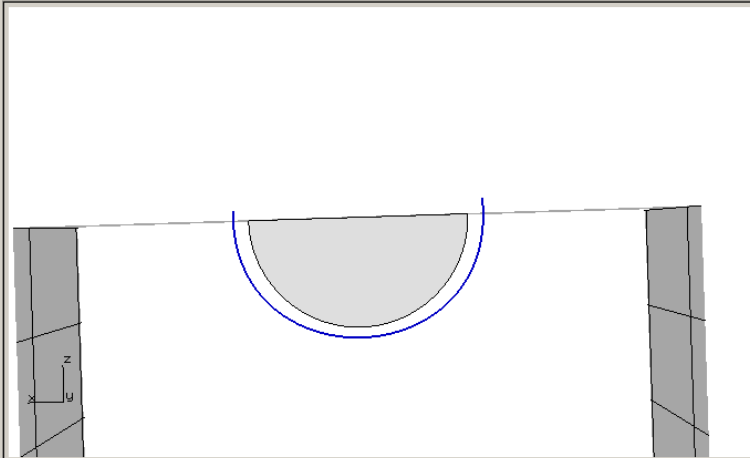
γ_{II} : γ_{III} :

FEM Load Steps

Step	Sub	Load Mult	Temp Mult	Temp Offset
<input type="text" value="1"/>	<input type="text" value=""/>	<input type="text" value="1"/>	<input type="text" value="1"/>	<input type="text" value="0"/>

Specify median extension and fitting parameters

Crack Growth



Display

- Markers
- Vectors
- Polygons
- Text
- Mesh
- Crk Geom

(reset) (d)
(crack)

Crack Extension

extension: scale node:

number of cycles:

elapsed time:

start cycle: start time:

Estimated Increments

estimated cycles: end cycle:

estimated time: end time:

Marker size: big med small

Front Fitting Options

KinkAngle/Extension Poly Fit polynomial order:

Fixed Order Poly Through Points extrapolate (%):

Multiple Poly Through Points ignore n end points:

Hermitian Closed Poly multiple poly ratio:

Cubic Spline moving poly range:

Moving Polynomial

No Smoothing

Allow fit adjustment

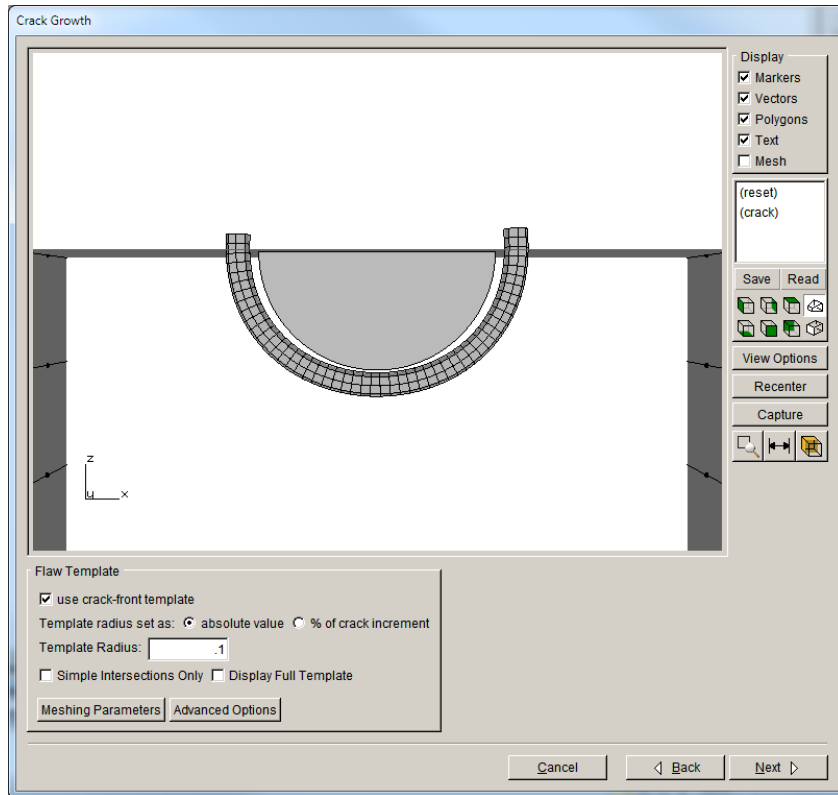
Kink Angle:

Extension:

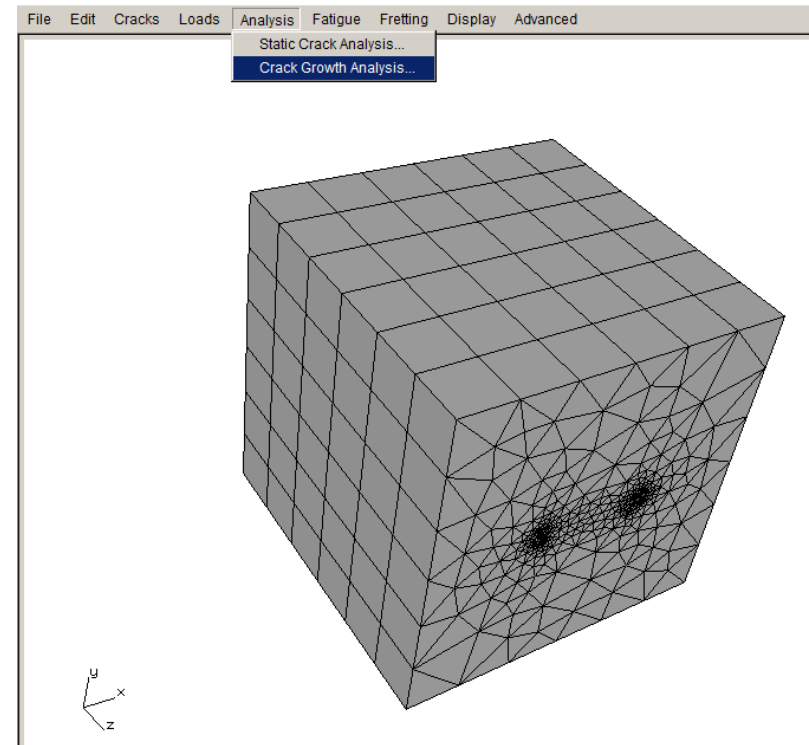
Save .ftt and .crk files

FRANC3D Tutorial 1 – Manual Crack Growth

Specify Template Mesh



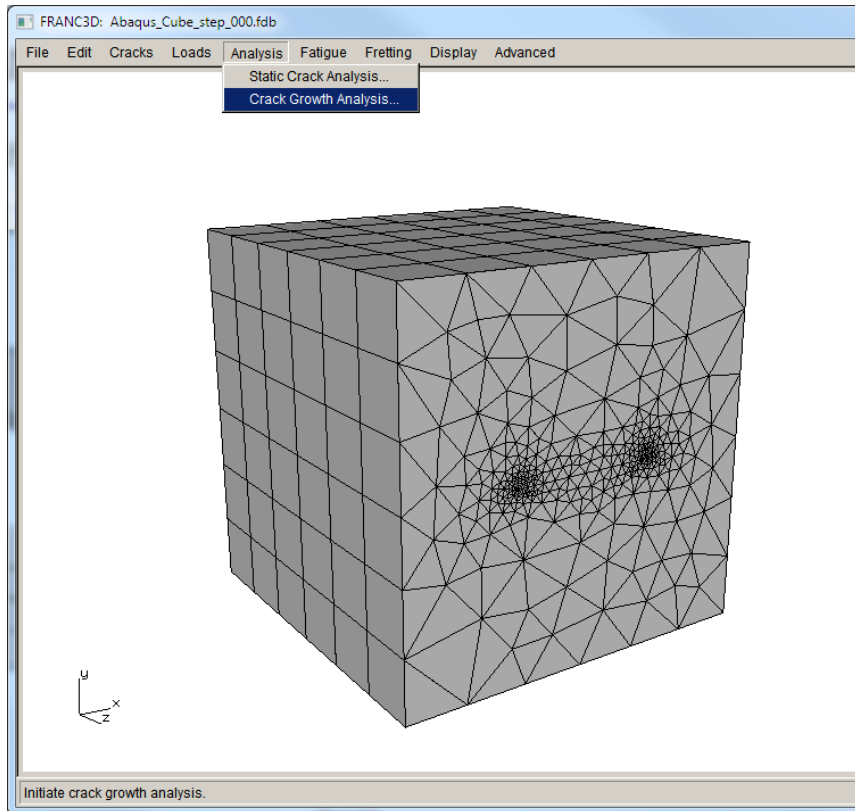
Resulting remeshed model
is ready for analysis.



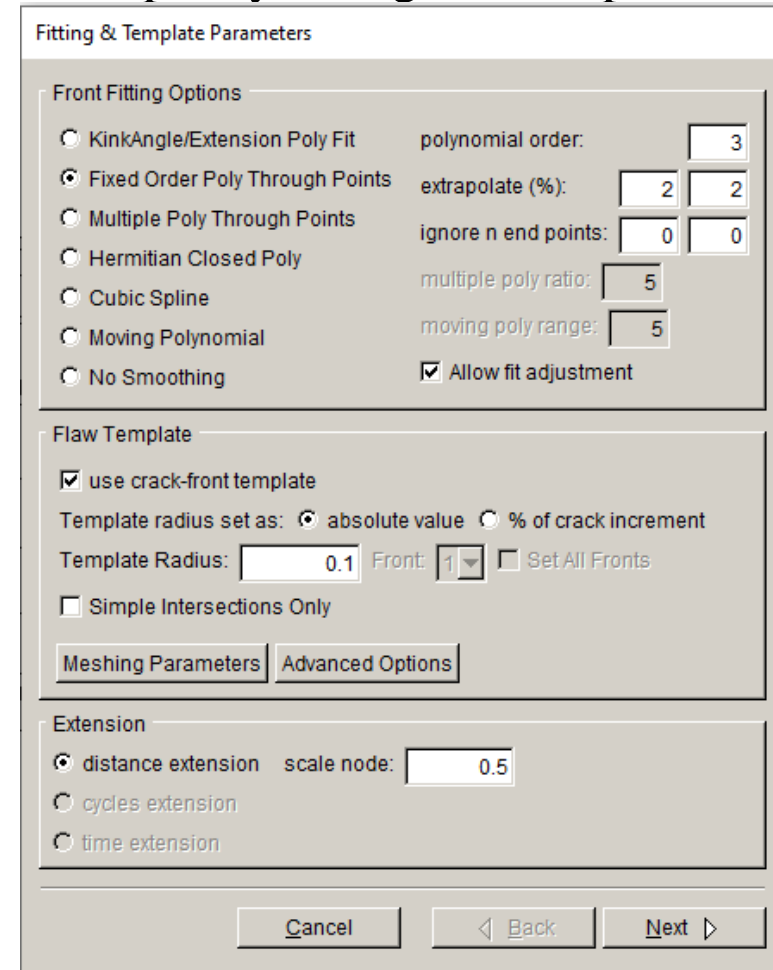
Demo/Hands On (Homework): Automated Crack Growth

FRANC3D Tutorial 1 – Automatic Crack Growth

From FRANC3D menu, select **Analysis - Crack Growth Analysis**



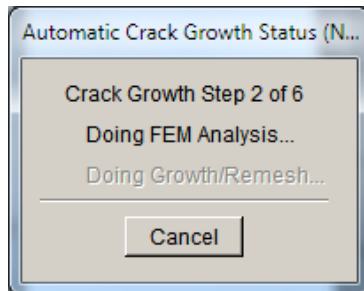
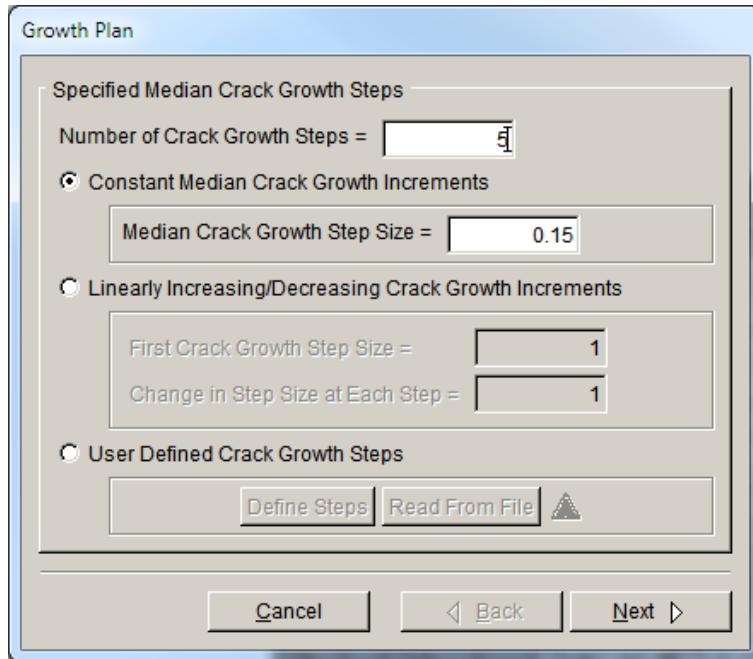
Re-specify fitting and template.



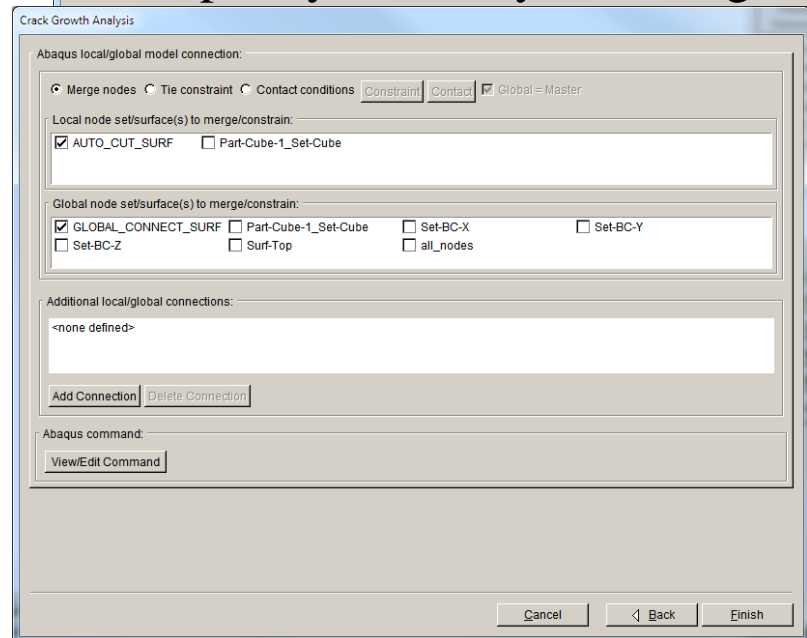
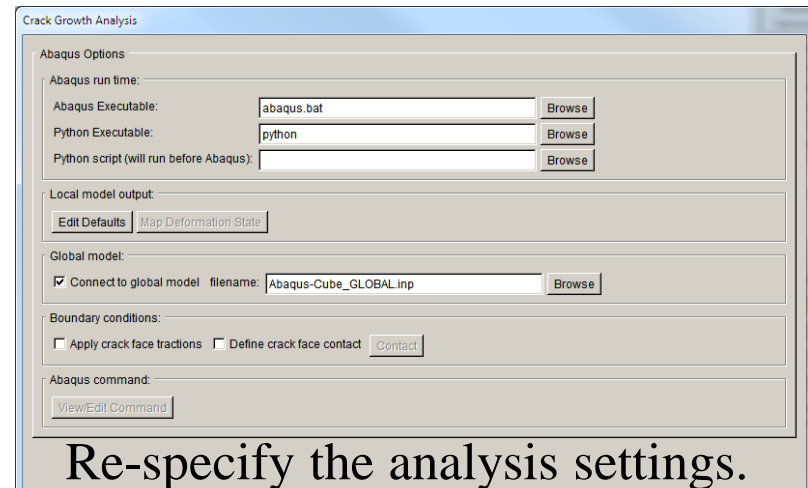
The crack growth rules and parameters set previously are used; this assumes static crack growth was done previously.

FRANC3D Tutorial 1 – Automatic Crack Growth

Specify number of growth steps.



Wait for the growth steps to be completed.



End Part 9