FRANC3D Training Workshop: Part 2

Introduction to Fracture Mechanics Analysis

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Workshop Agenda

- Part 1: Introduction to Fatigue and Damage Tolerance
- Part 2: Introduction to Fracture Mechanics Analysis
- Part 3: Introduction to FRANC3D
- Part 4: FRANC3D User Interface
- Part 5: Finite Element (FE) Model Import
- Part 6: Crack Insertion
- Part 7: Static Crack Analysis & SIF Computation
- Part 8: SIFs from FE Analysis
- Part 9: Crack Growth
- Part 10: SIF History & Fatigue Life
- Part 11: Miscellaneous Topics

Introduction to Fracture Mechanics Analysis

- What is Fracture Mechanics?
- Linear Elastic Fracture Mechanics
- Continuum Fracture Modes
- 2D Crack Front Stress and Displacement Fields
- Definition of Stress Intensity Factor
- So Why is the Stress Intensity Factor so Important?
- Concept of K-Dominance: When is LEFM Applicable?
- Energy Release Rate

What is Fracture Mechanics?

The ***classical*** objective of fracture mechanics is the determination of the rate of change of the shape of an ***existing*** crack. Will it propagate under given loading and environmental conditions, and, if it does propagate, at what rate, and into what configuration?

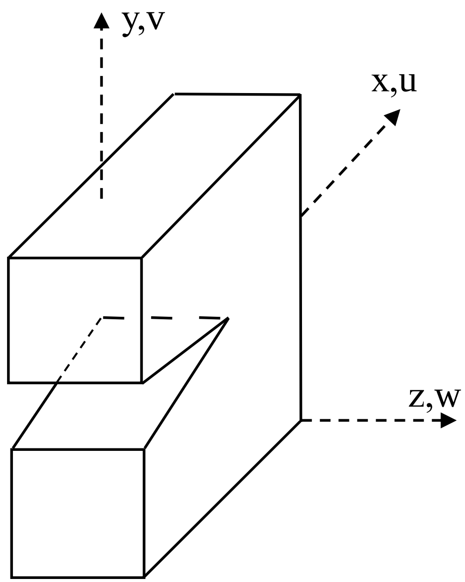
The corresponding *analytical computational* requirement has been to obtain the fields – displacement, strain, stress, and energy – from which the driving force for crack propagation might be extracted.

The corresponding *experimental* requirement has been to measure the resisting force against crack propagation, and to observe and measure configuration change and rate of growth.

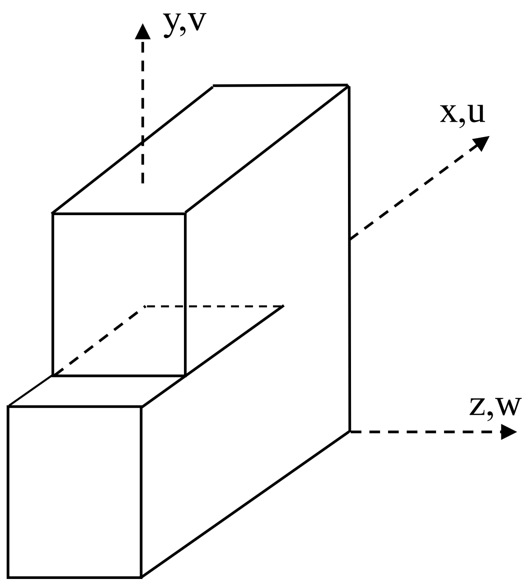
Linear Elastic Fracture Mechanics

- Review some basic elements of LEFM to prepare for computational implementation:
  - Crack front stress and displacement fields
  - Stress Intensity Factor
  - T-Stress
  - Energy Release Rate

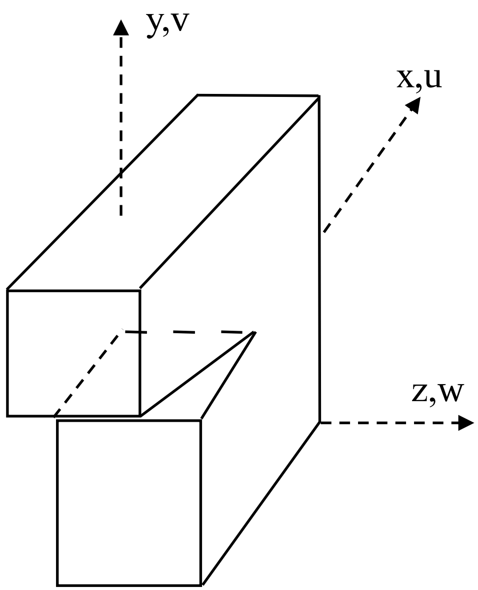
Continuum Fracture Modes



Mode I



Mode II



Mode III

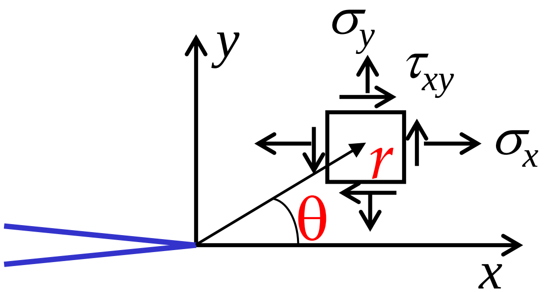
Basic modes of crack displacement; **positive sense** shown for each:

Mode I = crack opening

Mode II = in-plane sliding

Mode III = anti-plane tearing

2D Crack Front Stress and Displacement Fields



Williams **(1957) expansion of crack tip stress and displacement fields:**

Mode I

$$\sigma_x = \sum_{n=1}^{\infty} \frac{n}{2} r^{\frac{n}{2}-1} a_n^I \left[\left(2 + \frac{n}{2} + (-1)^n \right) \cos(\frac{n}{2}-1)\theta - (\frac{n}{2}-1)\cos(\frac{n}{2}-3)\theta \right]$$

$$\sigma_y = \sum_{n=1}^{\infty} \frac{n}{2} r^{\frac{n}{2}-1} a_n^I \left[\left(2 - \frac{n}{2} - (-1)^n \right) \cos(\frac{n}{2}-1)\theta + (\frac{n}{2}-1)\cos(\frac{n}{2}-3)\theta \right]$$

$$\tau_{xy} = \sum_{n=1}^{\infty} \frac{n}{2} r^{\frac{n}{2}-1} a_n^I \left[(\frac{n}{2}-1)\sin(\frac{n}{2}-3)\theta - \left(\frac{n}{2} + (-1)^n \right) \sin(\frac{n}{2}-1)\theta \right]$$

$$u = \sum_{n=1}^{\infty} \frac{r^{\frac{n}{2}}}{2\mu} a_n^I \left[\left(\kappa + \frac{n}{2} + (-1)^n \right) \cos\frac{n}{2}\theta - \frac{n}{2}\cos(\frac{n}{2}-2)\theta \right]$$

$$v = \sum_{n=1}^{\infty} \frac{r^{\frac{n}{2}}}{2\mu} a_n^I \left[\left(\kappa - \frac{n}{2} - (-1)^n \right) \sin\frac{n}{2}\theta + \frac{n}{2}\sin(\frac{n}{2}-2)\theta \right]$$

$\sigma_z = 0$
plane stress

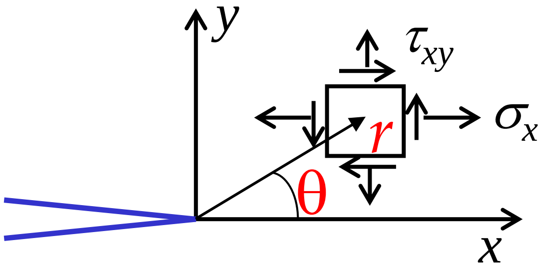
$\sigma_z = \nu(\sigma_x + \sigma_y)$
plane strain

$\tau_{xz} = \tau_{yz} = 0$

and where $\mu = G$
and $\kappa = 3-4\nu$, plane stress
$(3-\nu)/(1+\nu)$, plane strain

2D Crack Front Stress and Displacement Fields

Williams (1957) expansion of crack tip stress and displacement fields:



Mode II

$$\sigma_x = -\sum_{n=1}^{\infty} \frac{n}{2} r^{\frac{n}{2}-1} a_n^{II} \left[\left(2 + \frac{n}{2} - (-1)^n \right) \sin(\frac{n}{2}-1)\theta - (\frac{n}{2}-1)\sin(\frac{n}{2}-3)\theta \right]$$

$$\sigma_y = -\sum_{n=1}^{\infty} \frac{n}{2} r^{\frac{n}{2}-1} a_n^{II} \left[\left(2 - \frac{n}{2} + (-1)^n \right) \sin(\frac{n}{2}-1)\theta + (\frac{n}{2}-1)\sin(\frac{n}{2}-3)\theta \right]$$

$$\tau_{xy} = \sum_{n=1}^{\infty} \frac{n}{2} r^{\frac{n}{2}-1} a_n^{II} \left[(\frac{n}{2}-1)\cos(\frac{n}{2}-3)\theta - \left(\frac{n}{2} - (-1)^n \right) \cos(\frac{n}{2}-1)\theta \right]$$

$$\mathsf{u} = -\sum_{n=1}^{\infty} \frac{r^{\frac{n}{2}}}{2\mu} a_n^{II} \left[\left(\kappa + \frac{n}{2} - (-1)^n \right) \sin\frac{n}{2}\theta - \frac{n}{2}\sin(\frac{n}{2}-2)\theta \right]$$

$$\mathsf{v} = \sum_{n=1}^{\infty} \frac{r^{\frac{n}{2}}}{2\mu} a_n^{II} \left[\left(\kappa - \frac{n}{2} + (-1)^n \right) \cos\frac{n}{2}\theta + \frac{n}{2}\cos(\frac{n}{2}-2)\theta \right]$$

$$\sigma_z = 0$$

plane stress

$$\sigma_z = \nu\left(\sigma_x + \sigma_y\right)$$

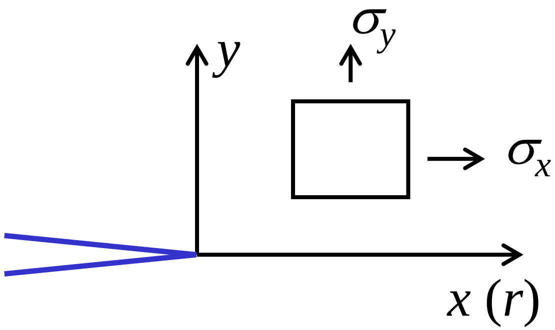
plane strain

$$\tau_{xz} = \tau_{yz} = 0$$

where μ = G
and κ = 3–4ν, plane stress
(3–ν)/(1+ν), plane strain

Crack Front Stress and Displacement Fields

Example of Expansion
Along Crack Line, x = r, Mode I



$$\sigma_x = \frac{a_1}{\sqrt{r}} + 4a_2 + 3a_3\sqrt{r} + 8a_4 r + 5a_5 r^{3/2} + \cdots$$

$$\sigma_y = \frac{a_1}{\sqrt{r}} + 3a_3\sqrt{r} + 5a_5 r^{3/2} + \cdots$$

First (leading), singular term, a_1: contains the *stress intensity factor*

Second term, a_2: contains the *T*-stress

Third term, a_3: leading higher order term (note: non-polynomial!)

Definition of Stress Intensity Factor and T-stress from these Fields

Neglecting all but the first singular term of this stress field results in the formal definition of the stress intensity factor:

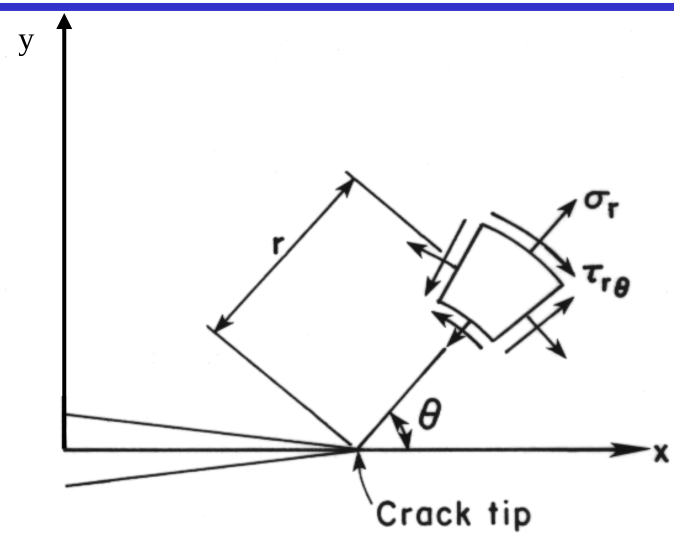
$$K_I = \lim_{r \to 0} \sigma_{yy} \sqrt{2 \pi r}$$

$$K_{II} = \lim_{r \to 0} \tau_{xy} \sqrt{2 \pi r}$$

$$K_{III} = \lim_{r \to 0} \tau_{yz} \sqrt{2 \pi r}$$

The T-stress is the constant stress acting parallel to the crack direction.

In Cylindrical Coordinates, to 2nd Term



$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left[K_I \cos^2\frac{\theta}{2} - \frac{3}{2}K_{II}\sin\theta\right] + \frac{T}{2}(1-\cos 2\theta)$$

$$\sigma_{rr} = \frac{1}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left[K_I\left(1+\sin^2\frac{\theta}{2}\right) + \frac{3}{2}K_{II}\sin\theta - 2K_{II}\tan\frac{\theta}{2}\right] + \frac{T}{2}(1+\cos 2\theta)$$

$$\sigma_{r\theta} = \frac{1}{2\sqrt{2\pi r}} \cos\frac{\theta}{2}\left[K_I\sin\theta + K_{II}(3\cos\theta - 1)\right] - \frac{T}{2}\sin 2\theta$$

Mode III Fields, Plane Strain

$$\tau_{xz} = \frac{K_{III}}{(2\pi r)^{1/2}} \sin\frac{\theta}{2} \qquad \tau_{yz} = \frac{K_{III}}{(2\pi r)^{1/2}} \cos\frac{\theta}{2}$$

$$\sigma_x = \sigma_y = \sigma_z = \tau_{xz} = 0$$

$$w = \frac{K_{III}}{G}\left[\frac{2r}{\pi}\right]^{1/2} \sin\frac{\theta}{2}$$

$$\mathrm{u} = \mathrm{v} = 0$$

for plane stress, let $\nu = \frac{\nu}{1+\nu}$

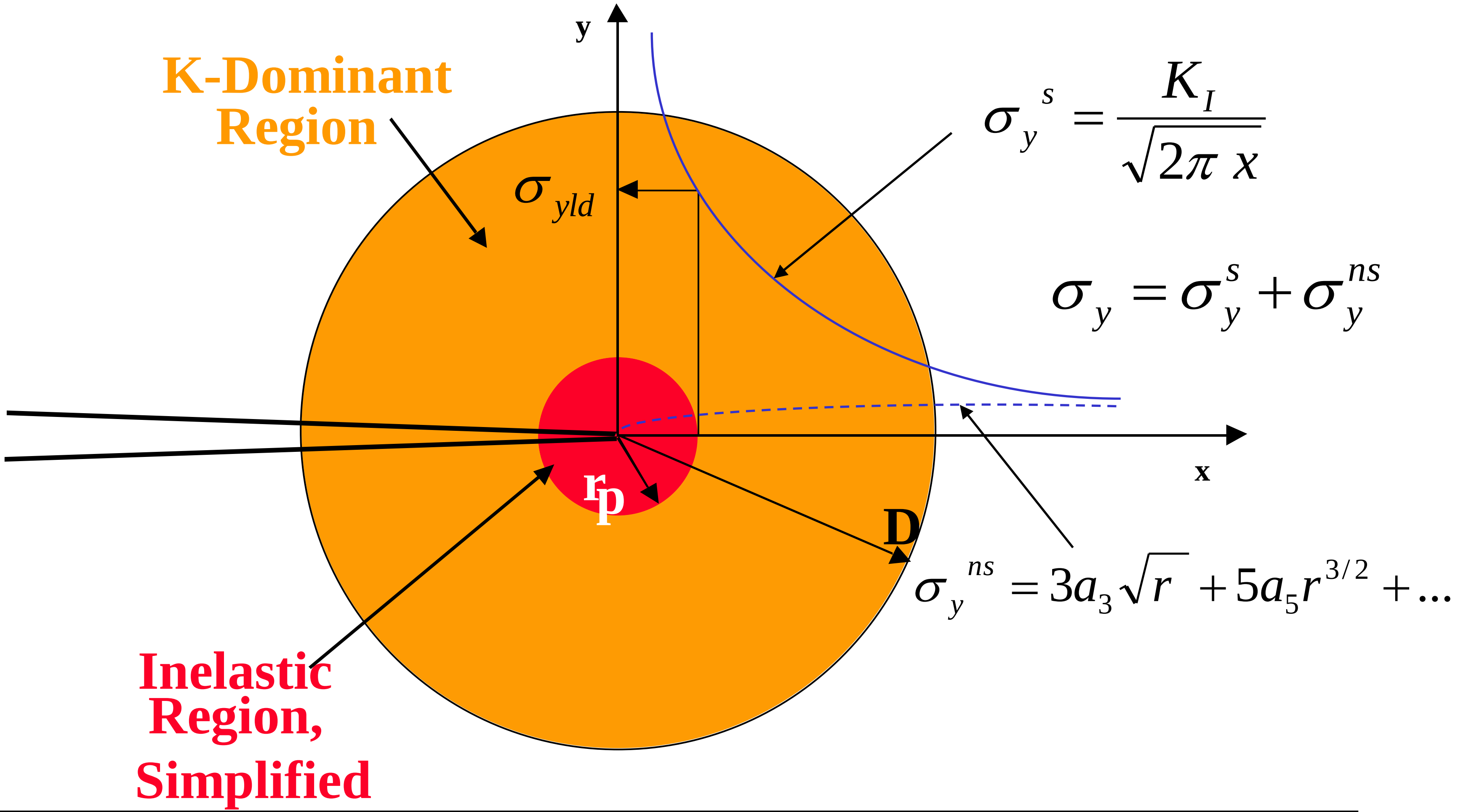
So Why is the Stress Intensity Factor so Important?

- Under conditions of small-scale yielding, all crack front fields are dominated (controlled) by the stress intensity factor.

- Therefore, all crack behavior:
  - Stability—will the crack tip move?
  - Trajectory– in what direction?
  - Rate– how fast?

  is controlled by the stress intensity factor and maybe T-stress

Concept of K-Dominance: When is LEFM Applicable?



If r_p << D, K_I still controls fracture process.

Energy Release Rate

Recall that in LEFM, energy release rate (crack driving *force*) is a dual of stress intensity. For example, in Mode I:

$$G_I = \frac{K_I^2}{E'}$$

where

$$E' = E \qquad \text{for plane stress}$$

$$E' = \frac{E}{(1-\nu^2)} \quad \text{for plane strain}$$

We will first concentrate on computing stress intensity factors, then later, energy release rates.

End Part 2