

FRANC3D Training Workshop: Part 2

Introduction to Fracture Mechanics Analysis

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Workshop Agenda

- Part 1: Introduction to Fatigue and Damage Tolerance
- **Part 2: Introduction to Fracture Mechanics Analysis**
- Part 3: Introduction to FRANC3D
- Part 4: FRANC3D User Interface
- Part 5: Finite Element (FE) Model Import
- Part 6: Crack Insertion
- Part 7: Static Crack Analysis & SIF Computation
- Part 8: SIFs from FE Analysis
- Part 9: Crack Growth
- Part 10: SIF History & Fatigue Life
- Part 11: Miscellaneous Topics

Introduction to Fracture Mechanics

- What is Fracture Mechanics?
- Linear Elastic Fracture Mechanics
- Continuum Fracture Modes
- 2D Crack Front Stress and Displacement Fields
- Definition of Stress Intensity Factor
- So Why is the Stress Intensity Factor so Important?
- Concept of K-Dominance: When is LEFM Applicable?
- Energy Release Rate

What is Fracture Mechanics?

The ***classical*** objective of fracture mechanics is the determination of the rate of change of the shape of an ***existing*** crack. Will it propagate under given loading and environmental conditions, and, if it does propagate, at **what rate**, and into **what configuration**?

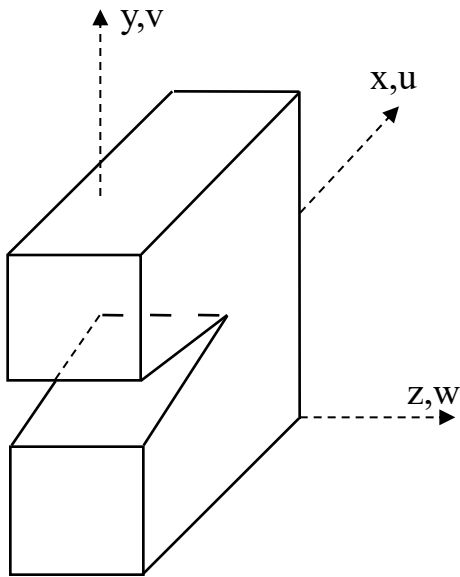
The corresponding ***analytical computational*** requirement has been to obtain the fields – displacement, strain, stress, and energy – from which the driving force for crack propagation might be extracted.

The corresponding ***experimental*** requirement has been to **measure the resisting force against crack propagation**, and to observe and measure configuration change and rate of growth.

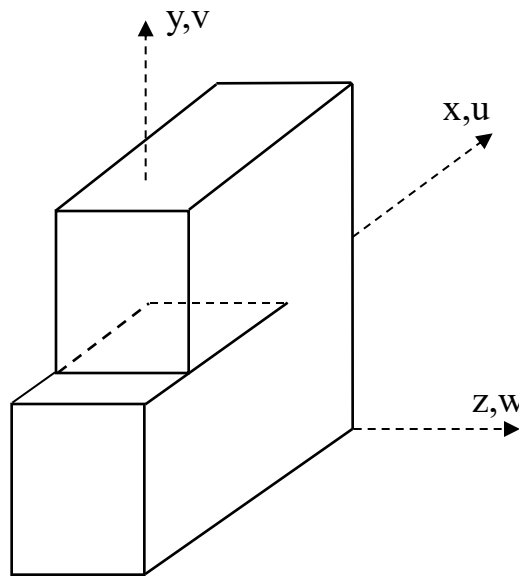
Linear Elastic Fracture Mechanics

- Review some basic elements of LEFM to prepare for computational implementation:
 - Crack front stress and displacement fields
 - Stress Intensity Factor
 - T-Stress
 - Energy Release Rate

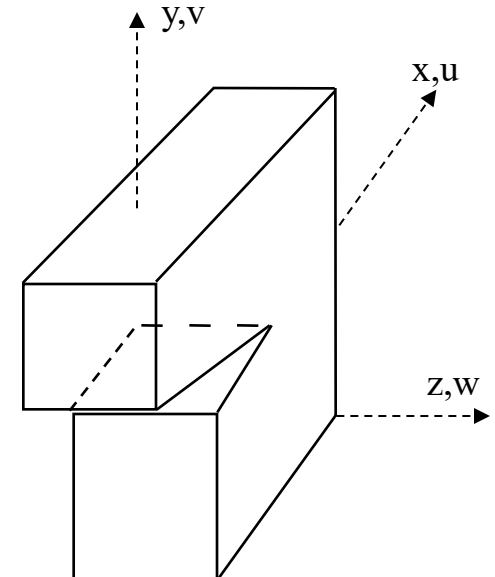
Continuum Fracture Modes



Mode I



Mode II



Mode III

Basic modes of crack displacement; **positive sense** shown for each:

Mode I = crack opening

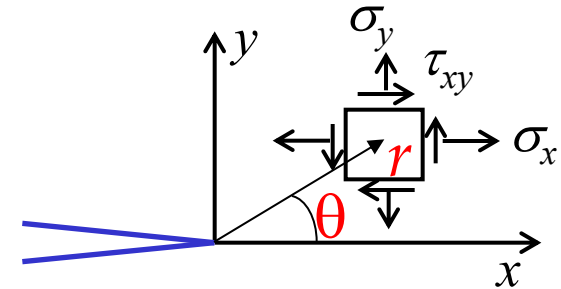
Mode II = in-plane sliding

Mode III = anti-plane tearing

2D Crack Front Stress and Displacement Fields

Williams (1957) expansion of crack tip stress and displacement fields:

Mode I



$$\sigma_x = \sum_{n=1}^{\infty} \frac{n}{2} r^{\frac{n}{2}-1} a_n^I \left[\left(2 + \frac{n}{2} + (-1)^n \right) \cos\left(\frac{n}{2}-1\right)\theta - \left(\frac{n}{2}-1\right) \cos\left(\frac{n}{2}-3\right)\theta \right]$$

$$\sigma_y = \sum_{n=1}^{\infty} \frac{n}{2} r^{\frac{n}{2}-1} a_n^I \left[\left(2 - \frac{n}{2} - (-1)^n \right) \cos\left(\frac{n}{2}-1\right)\theta + \left(\frac{n}{2}-1\right) \cos\left(\frac{n}{2}-3\right)\theta \right]$$

$$\tau_{xy} = \sum_{n=1}^{\infty} \frac{n}{2} r^{\frac{n}{2}-1} a_n^I \left[\left(\frac{n}{2}-1\right) \sin\left(\frac{n}{2}-3\right)\theta - \left(\frac{n}{2} + (-1)^n\right) \sin\left(\frac{n}{2}-1\right)\theta \right]$$

$$u = \sum_{n=1}^{\infty} \frac{r^{\frac{n}{2}}}{2\mu} a_n^I \left[\left(\kappa + \frac{n}{2} + (-1)^n \right) \cos\frac{n}{2}\theta - \frac{n}{2} \cos\left(\frac{n}{2}-2\right)\theta \right]$$

$$v = \sum_{n=1}^{\infty} \frac{r^{\frac{n}{2}}}{2\mu} a_n^I \left[\left(\kappa - \frac{n}{2} - (-1)^n \right) \sin\frac{n}{2}\theta + \frac{n}{2} \sin\left(\frac{n}{2}-2\right)\theta \right]$$

$\sigma_z = 0$
plane stress

$\sigma_z = \nu(\sigma_x + \sigma_y)$
plane strain

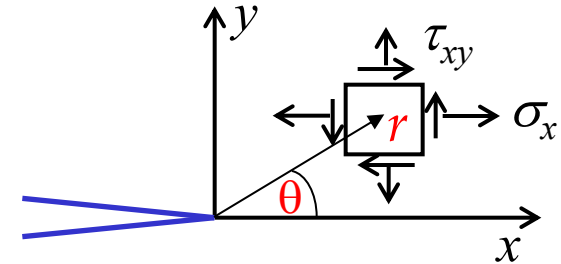
$$\tau_{xz} = \tau_{yz} = 0$$

where $\mu = G$
and $\kappa = 3-4\nu$, plane stress
($3-\nu$)/($1+\nu$), plane strain

2D Crack Front Stress and Displacement Fields

Williams (1957) expansion of crack tip stress and displacement fields:

Mode II



$$\sigma_x = -\sum_{n=1}^{\infty} \frac{n}{2} r^{\frac{n}{2}-1} a_n^{\text{II}} \left[\left(2 + \frac{n}{2} - (-1)^n \right) \sin\left(\frac{n}{2}-1\right)\theta - \left(\frac{n}{2}-1\right) \sin\left(\frac{n}{2}-3\right)\theta \right]$$

$$\sigma_y = -\sum_{n=1}^{\infty} \frac{n}{2} r^{\frac{n}{2}-1} a_n^{\text{II}} \left[\left(2 - \frac{n}{2} + (-1)^n \right) \sin\left(\frac{n}{2}-1\right)\theta + \left(\frac{n}{2}-1\right) \sin\left(\frac{n}{2}-3\right)\theta \right]$$

$$\tau_{xy} = \sum_{n=1}^{\infty} \frac{n}{2} r^{\frac{n}{2}-1} a_n^{\text{II}} \left[\left(\frac{n}{2}-1\right) \cos\left(\frac{n}{2}-3\right)\theta - \left(\frac{n}{2}-(-1)^n\right) \cos\left(\frac{n}{2}-1\right)\theta \right]$$

$$u = -\sum_{n=1}^{\infty} \frac{r^{\frac{n}{2}}}{2\mu} a_n^{\text{II}} \left[\left(\kappa + \frac{n}{2} - (-1)^n \right) \sin \frac{n}{2} \theta - \frac{n}{2} \sin\left(\frac{n}{2}-2\right)\theta \right]$$

$$v = \sum_{n=1}^{\infty} \frac{r^{\frac{n}{2}}}{2\mu} a_n^{\text{II}} \left[\left(\kappa - \frac{n}{2} + (-1)^n \right) \cos \frac{n}{2} \theta + \frac{n}{2} \cos\left(\frac{n}{2}-2\right)\theta \right]$$

Part 2

$\sigma_z = 0$
plane stress

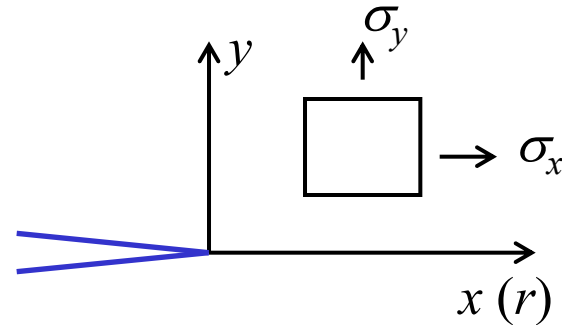
$\sigma_z = \nu(\sigma_x + \sigma_y)$
plane strain

$$\tau_{xz} = \tau_{yz} = 0$$

where $\mu = G$
and $\kappa = 3-4\nu$, plane stress
 $(3-\nu)/(1+\nu)$, plane strain

Crack Front Stress and Displacement Fields

Example of expansion along
crack line, $x = r$, Mode I



$$\sigma_x = \frac{a_1}{\sqrt{r}} + 4a_2 + 3a_3\sqrt{r} + 8a_4r + 5a_5r^{3/2} + \dots$$

$$\sigma_y = \frac{a_1}{\sqrt{r}} + 3a_3\sqrt{r} + 5a_5r^{3/2} + \dots$$

First (leading), singular term, a_1 : contains the *stress intensity factor*

Second term, a_2 : contains the *T-stress*

Third term, a_3 : leading higher order term (note: non-polynomial!)

Definition of Stress Intensity Factor and T-stress from these Fields

Neglecting all but the first singular term of this stress field results in the formal definition of the stress intensity factor:

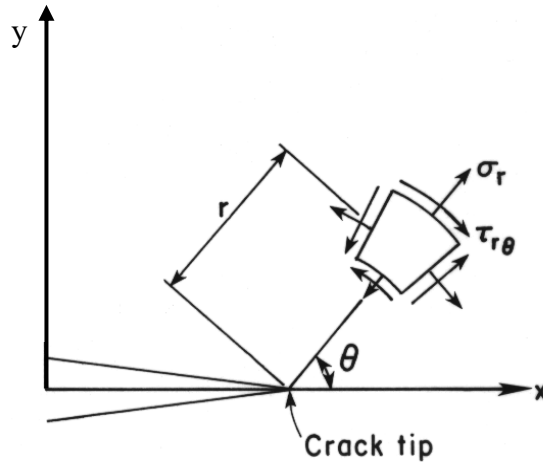
$$K_I = \lim_{r \rightarrow 0} \sigma_{yy} \sqrt{2 \pi r}$$

$$K_{II} = \lim_{r \rightarrow 0} \tau_{xy} \sqrt{2 \pi r}$$

$$K_{III} = \lim_{r \rightarrow 0} \tau_{yz} \sqrt{2 \pi r}$$

T-stress is the constant stress acting parallel to the crack direction.

In Cylindrical Coordinates, to 2nd Term



$\longleftrightarrow$ T -stress $\longleftrightarrow$

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[K_I \cos^2 \frac{\theta}{2} - \frac{3}{2} K_{II} \sin \theta \right] + \frac{T}{2} (1 - \cos 2\theta)$$

$$\sigma_{rr} = \frac{1}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[K_I \left(1 + \sin^2 \frac{\theta}{2} \right) + \frac{3}{2} K_{II} \sin \theta - 2K_{II} \tan \frac{\theta}{2} \right] + \frac{T}{2} (1 + \cos 2\theta)$$

$$\sigma_{r\theta} = \frac{1}{2\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[K_I \sin \theta + K_{II} (3 \cos \theta - 1) \right] - \frac{T}{2} \sin 2\theta$$

Mode III Fields, Plane Strain

$$\tau_{xz} = \frac{K_{III}}{(2\pi r)^{1/2}} \sin \frac{\theta}{2}$$

$$\tau_{yz} = \frac{K_{III}}{(2\pi r)^{1/2}} \cos \frac{\theta}{2}$$

$$\sigma_x = \sigma_y = \sigma_z = \tau_{xz} = 0$$

$$w = \frac{K_{III}}{G} \left[\frac{2r}{\pi} \right]^{1/2} \sin \frac{\theta}{2}$$

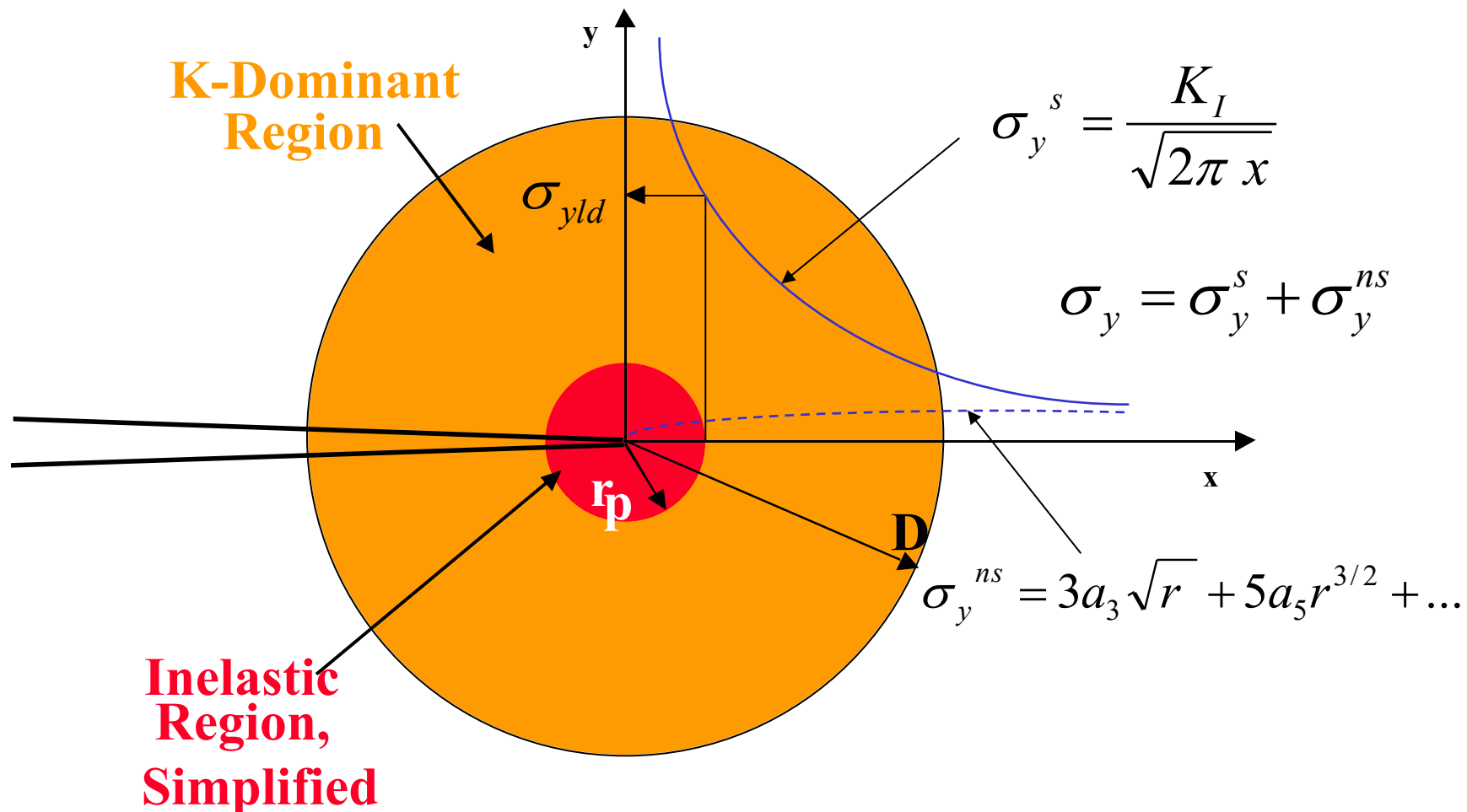
$$u = v = 0$$

for plane stress, let $\nu = \frac{\nu}{1+\nu}$

So why is Stress Intensity Factor so Important?

- Under conditions of small-scale yielding, all crack front fields are dominated (controlled) by the stress intensity factor.
- Therefore, **all crack behavior**:
 - Stability—will the crack tip move?
 - Trajectory— in what direction?
 - Rate— how fast?**is controlled by the stress intensity factor and maybe T-stress**

Concept of K-Dominance: When is LEFM Applicable?



If $r_p \ll D$, K_I still controls fracture process.

Energy Release Rate

Recall that in LEFM, energy release rate (*crack driving force*) is a dual of stress intensity. For example, in Mode I:

$$G_I = \frac{K_I^2}{E'}$$

where

$$E' = E \quad \text{for plane stress}$$

$$E' = \frac{E}{(1-\nu^2)} \quad \text{for plane strain}$$

We will first concentrate on computing stress intensity factors, then later, energy release rates.

End Part 2