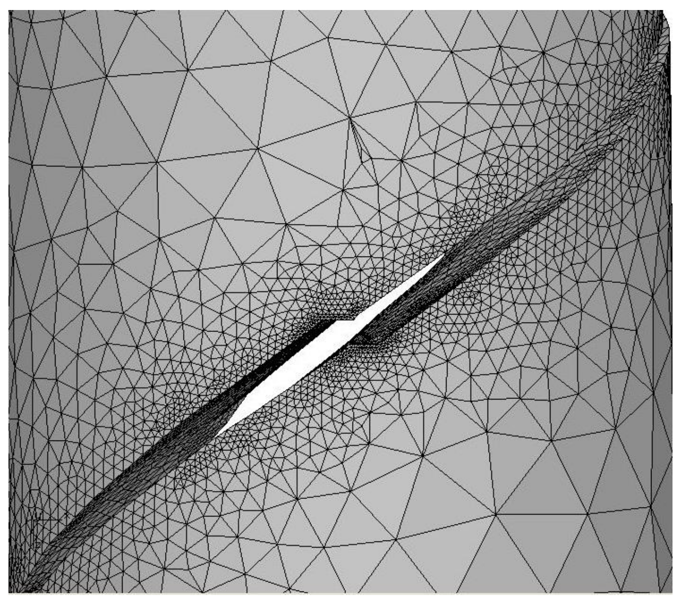
FRANC3D

Benchmark Examples



Version 8.4

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1 Introduction

Benchmark examples and comparisons between analytical or handbook and FRANC3D stress intensity factors (SIF) are provided herein.

FRANC3D SIFs are computed using the M-integral approach, computed at crack front element mid-side nodes. Comparisons between the M-integral, VCCT and DC SIFs are included in the User's Guide, Chapters 11 and 12.

2 Interior Penny Shaped Crack in a Rectilinear Bar (Sneddon Solution)

An internal circular (penny-shape) crack is simulated in a rectilinear bar, Fig 2.1. The elastic modulus is 10,000 and Poisson's ratio (Nu) is 0.0. The bar is constrained with simple supports. Unit traction is applied to the (left and right) end surfaces as indicated by the red arrows. ANSYS is used for the initial elastic stress analysis of the uncracked model, and the mesh information is archived to a *.cdb* file for use in FRANC3D.

We import the ANSYS FE model and use FRANC3D's submodeling tool to extract a local portion from the middle section of the bar. The crack is centered in the model and has a radius of 0.1 units, Fig 2.2.

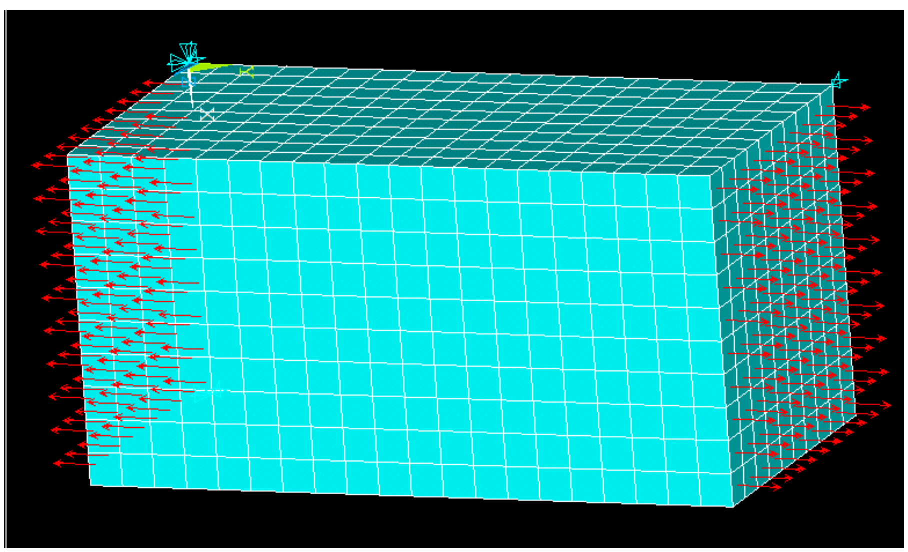


Figure 2.1 Rectilinear bar in ANSYS. It is 10 units long, 5 units wide and 5 units deep, with simple supports and unit uniform traction in the y-direction.

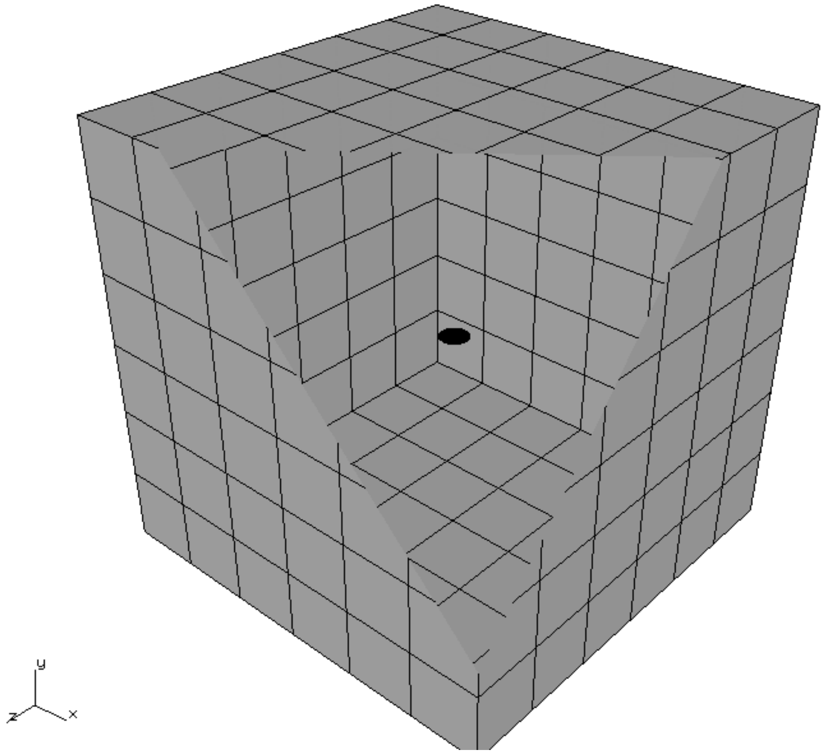


Figure 2.2 Internal penny-shaped crack with radius = 0.1 units.

The default crack front template radius is 0.01units, Fig 2.3, with 8 elements around the crack front and 3 rings of elements (quarter-point wedge with two rings of bricks). About 31,000 elements are used; this includes the elements in the global portion of the model, which is not remeshed.

One or two levels of mesh refinement can be used to study the accuracy of the SIFs. We start by increasing the number of rings of elements in the mesh template from 3 to 5, and at the same time, reduce the template element aspect ratio from 2 to 1. Increasing the number of rings reduces the size of the elements adjacent to the crack front while keeping the pyramid and tetrahedral elements the same distance away from the front. About 150,000 elements are generated.

An additional level of mesh refinement can be achieved by altering the FRANC3D advanced meshing parameters, Fig 2.4. Decreasing the values for the “volume factors” increases the mesh density. Using values of 0.65, 0.9, and 1.9 for the three factors, generates about 160,000 elements.

We could also switch from FRANC3D volume meshing to ABAQUS volume meshing, which will produce a different tetrahedral mesh in the region around the crack front.

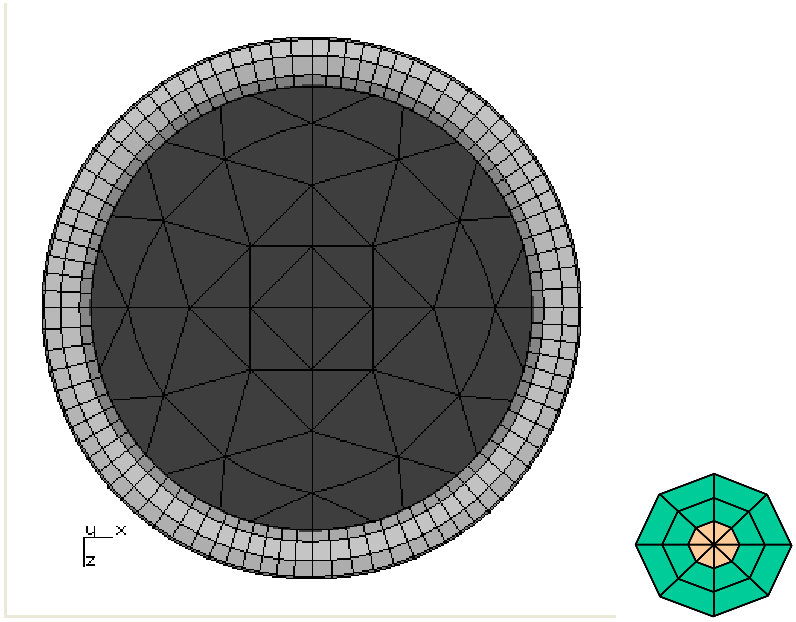


Figure 2.3. Crack surface and crack front template mesh, plan view.
The typical template cross-section is shown on the right.

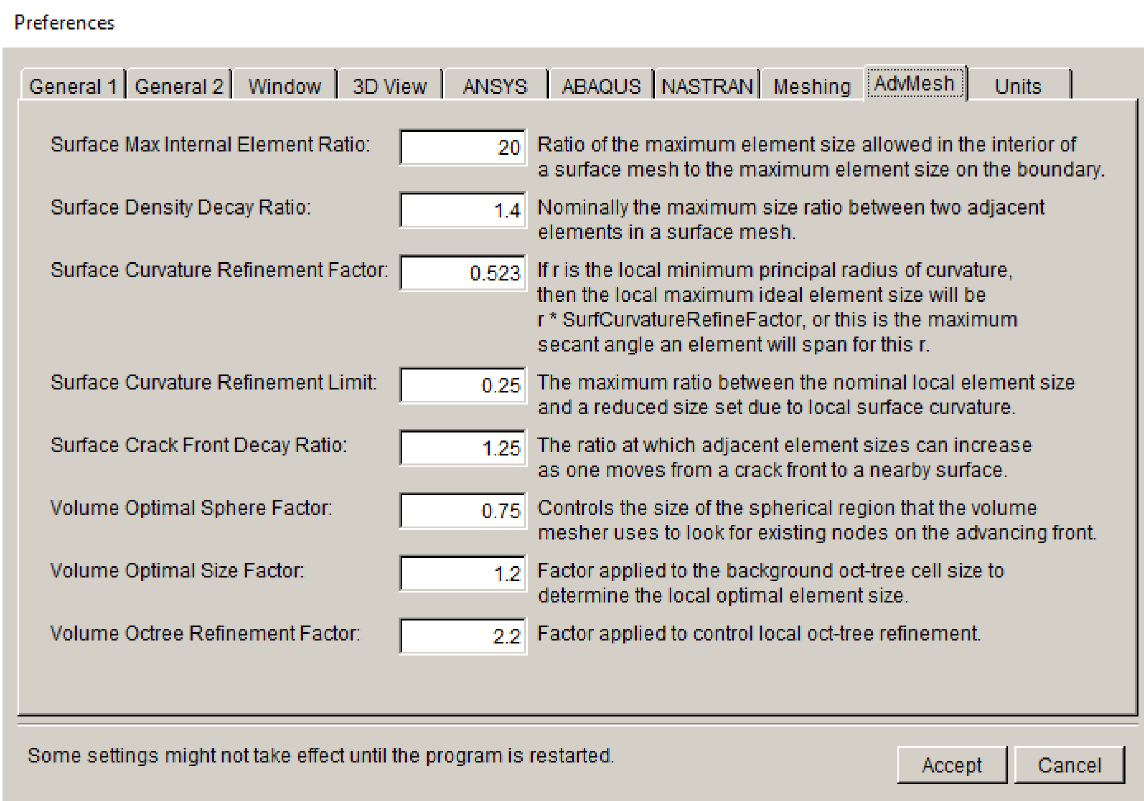


Figure 2.4. Advanced meshing parameters.

The resulting Mode I SIFs for the three analyses are shown in Fig 2.5. The % error (difference from the analytical solution) is shown in Fig 2.6.

The analytical solution for the Mode I SIF for this crack, in an infinite domain, was given by Sneddon (see Murakami, 1987) as:

$$K_I = 2\sigma\sqrt{\frac{a}{\pi}}$$

$$K_{II} = K_{III} = 0$$

The target value of K_I is 0.357 and is constant along the crack front.

The mean value for the default FRANC3D mesh is 0.356, and the mean difference from the analytical value is 0.25%. The mean value for the second case, where the template has 5 rings, is 0.355 and the mean difference in this case is 0.42%. For the third case, with the more refined mesh, the mean value is 0.3573 and the mean difference is 0.1%.

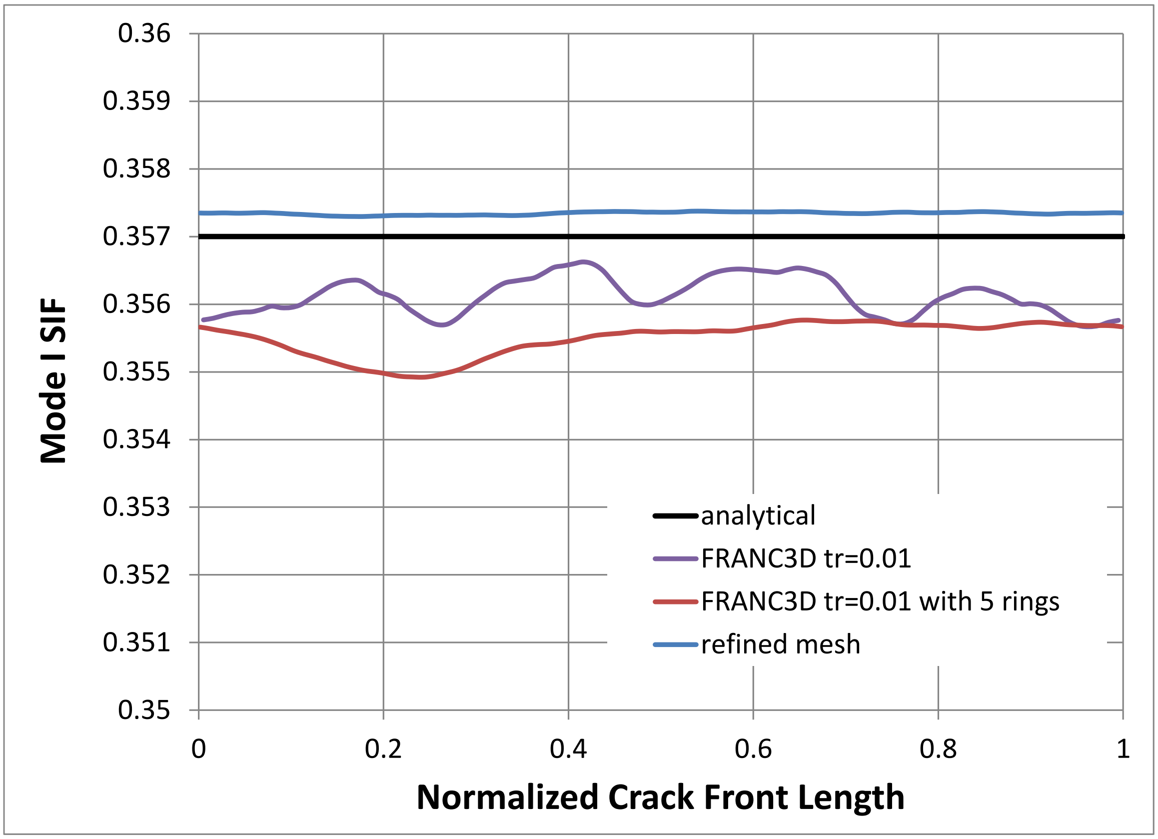


Figure 2.5 Mode I SIFs for the internal penny-shape crack.

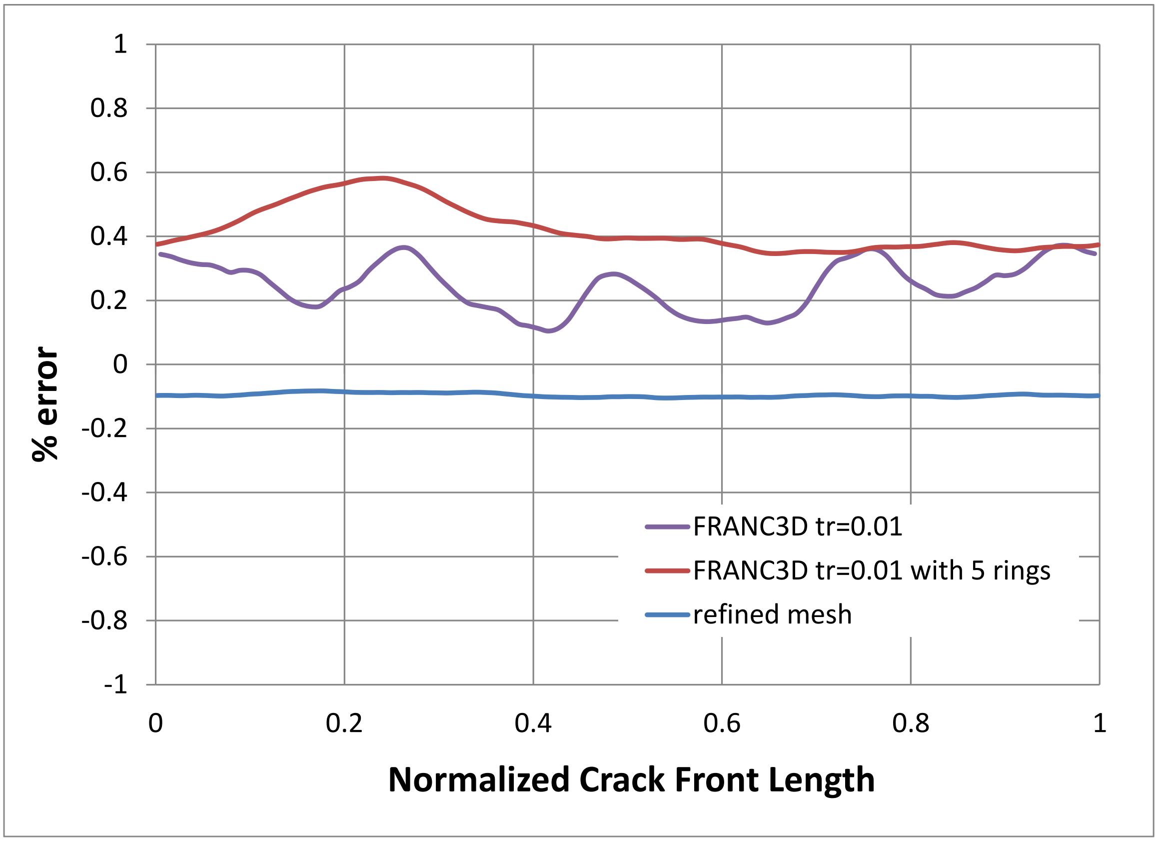


Figure 2.6 % error in Mode I SIFs for the internal penny-shape crack.

2.1 Crack Face Pressure (Traction)

FRANC3D allows one to apply crack face pressure or traction (CFT). The ANSYS FE model and crack geometry from Section 2.1 are used to compare the Mode I SIF solution for remote loading versus CFT. The only difference is that the far-field traction shown in Fig 2.1 is removed from the FE model, and instead, FRANC3D is used to apply a constant unit crack face pressure.

The template radius is 0.01 with 3 rings of elements and an aspect ratio of 2.

The Mode I SIFs are shown in Fig 2.7 and the % error (difference from reference solution) is shown in Fig 2.8. The average Mode I SIF is 0.355 and the average % error is 0.48. These SIF values are nearly identical to the corresponding SIFs for far-field loading.

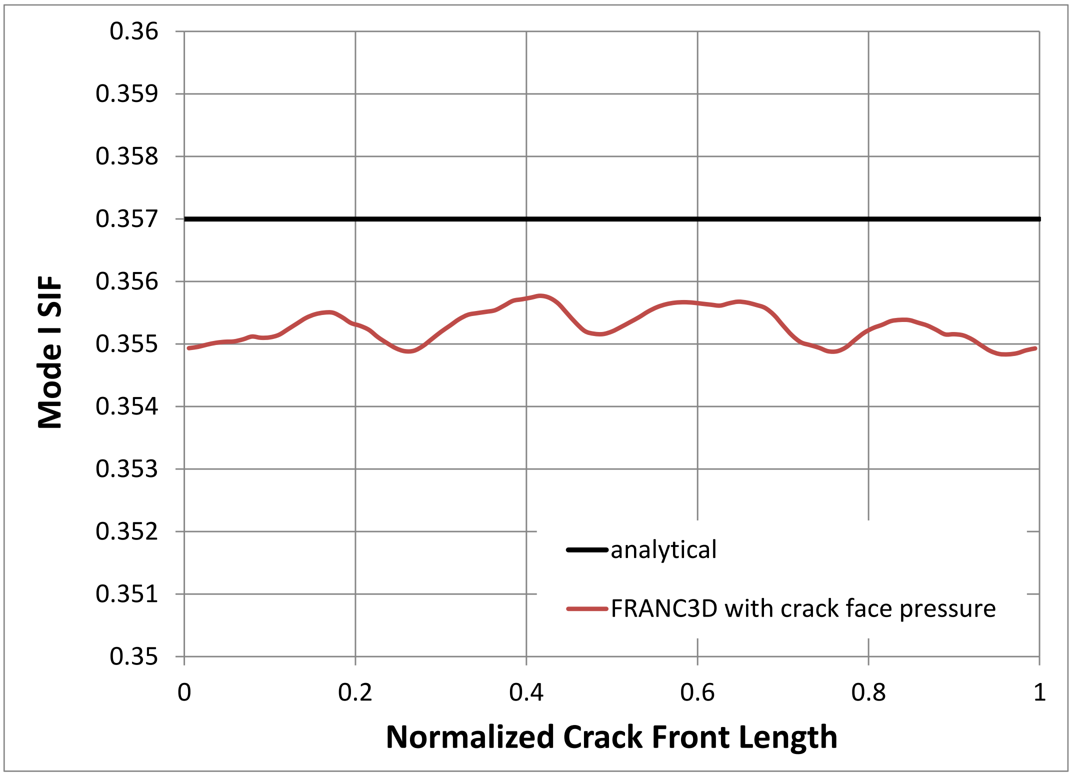


Figure 2.7 Mode I SIFs for the internal penny-shape crack with CFT.

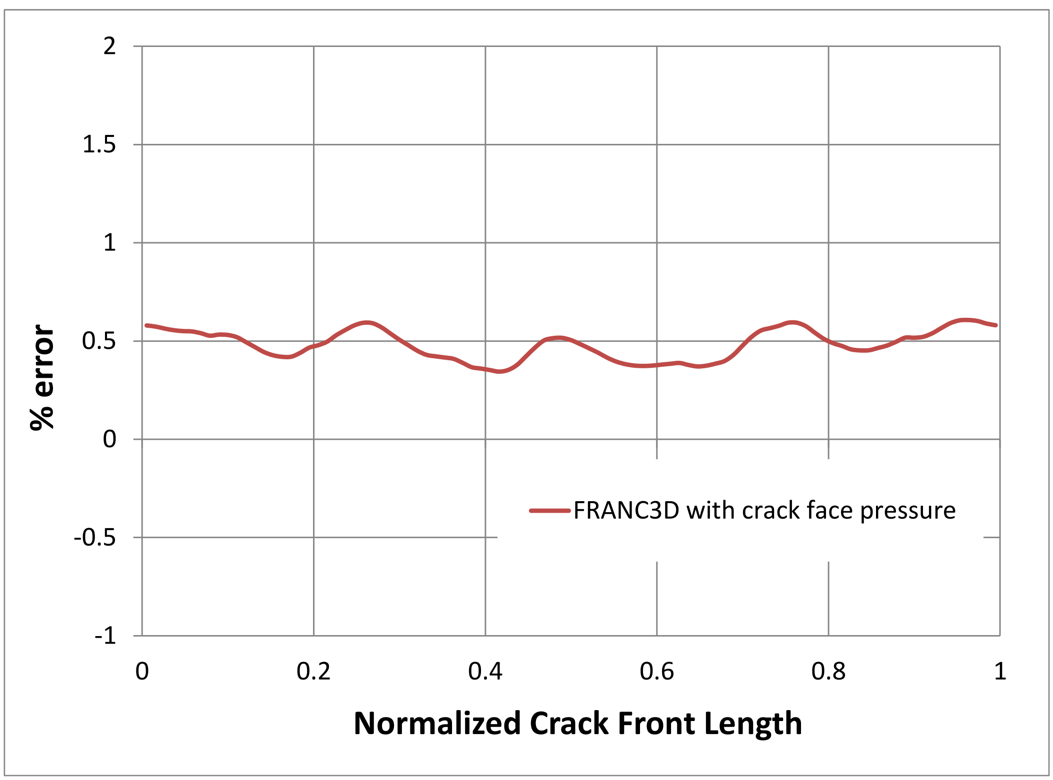


Figure 2.8 % error in Mode I SIFs for the internal penny-shape crack with CFT.

3 Internal, Inclined (45 degrees), Penny-Shaped Crack

A 10x5x5 block is used to model an infinite body containing a centered, internal, penny-shaped crack, with radius=0.125, and inclined at 45 degrees, Fig 3.1. The loading is far-field uniform tension equal to 1.0, and the material properties consist of an elastic modulus of 10,000 and a Poisson's ratio of 0.0. An ANSYS FE model is created and the corresponding *.cdb* file is used in FRANC3D; Fig 3.1 shows a local portion of the full model.

The default crack front template mesh is shown in Fig 3.2. The default template radius is 0.0125 units with 8 rings around the crack front and 3 rings of elements (quarter-point wedge and 2 rings of bricks). Based on the results in Section 2, we set the template radius to 0.01, increase the number of rings in the template from 3 to 5, and set the aspect ratio to 1. About 37,000 elements are generated for the entire model.

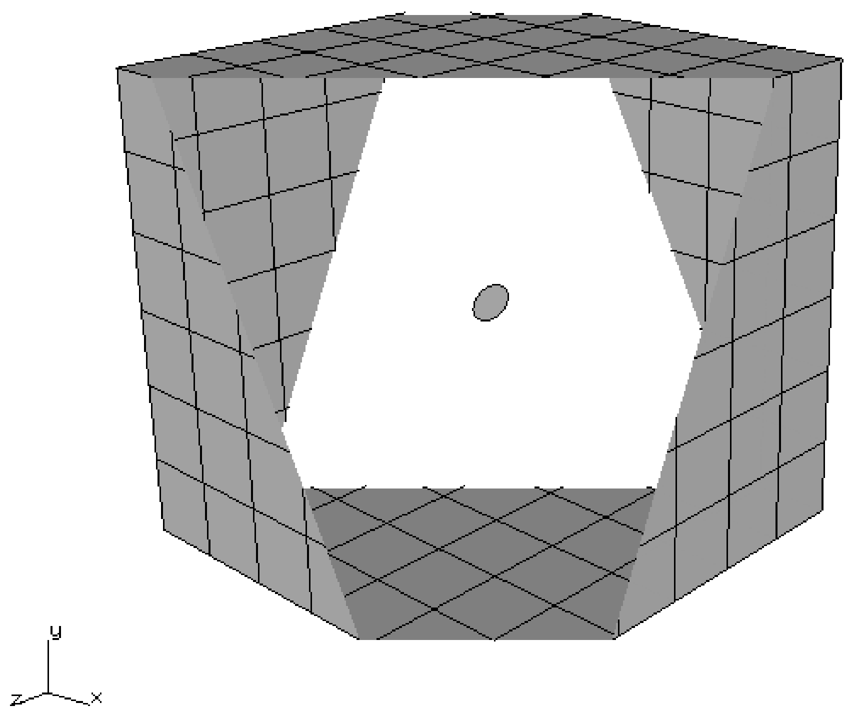


Figure 3.1 Internal, inclined at 45 degrees, penny-shaped crack in a rectilinear bar.

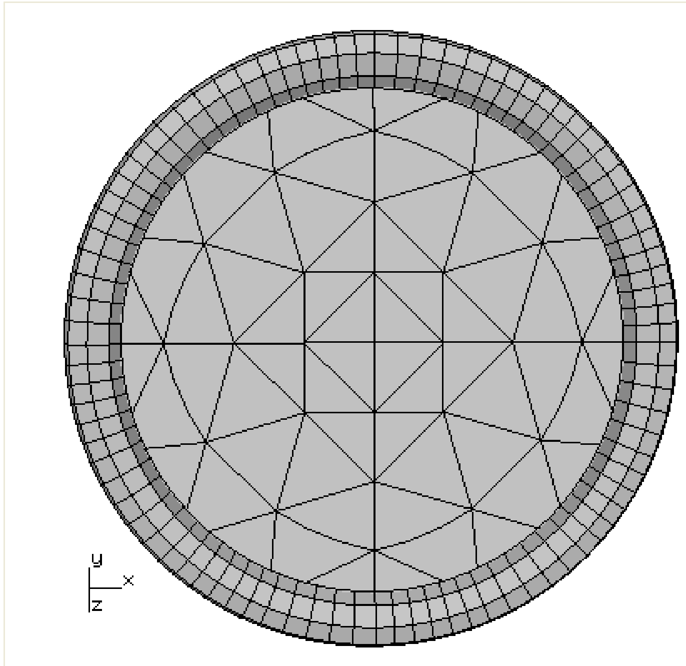


Figure 3.2 Default crack front template mesh.

The analytical values for Mode I, II and III SIFs for this configuration are available in Murakami (1987). Mode I SIF is constant at 0.200 along the crack front perimeter. The Mode II SIF is zero at the points A and B, Fig 3.3, and reaches a maximum (or minimum depending on the sign) at points C and D equal to 0.2 (or -0.2). Mode III SIF is zero at points C and D and reaches a maximum at points A and B equal to 0.2. The equations for the three modes of SIF are:

$$K_I = \sigma \sin^2(\gamma)\sqrt{\pi r}\,(2/\pi)$$
$$K_{II} = \sigma \sin(\gamma)\cos(\gamma)\sin(\theta)\sqrt{\pi r}\,(2/\pi)(2/(2-\nu))$$
$$K_{III} = \sigma \sin(\gamma)\cos(\gamma)\cos(\theta)\sqrt{\pi r}\,(2/\pi)(2/(2-\nu))(1-\nu)$$

where γ is the angle of inclination (here 45 degrees), θ is the position along the crack front, r is the crack radius.

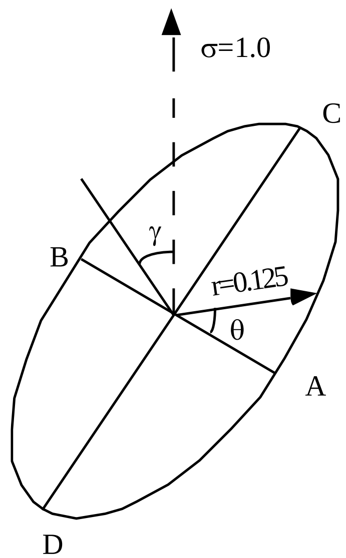


Figure 3.3 Inclined, penny-shaped crack subjected to uniform, remote tension.

Fig 3.4 shows all three SIF modes. The mean value of Mode I is 0.199. The average error in Mode I SIF is 0.32%. The maximum absolute Mode II and Mode III values are 0.199 and 0.199.

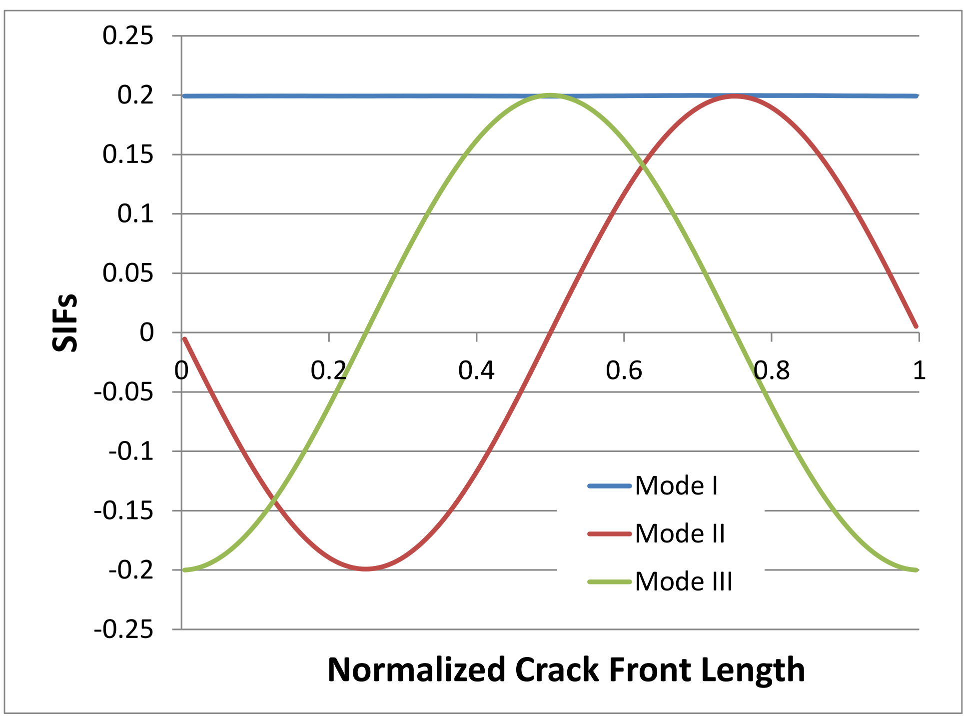


Figure 3.4 Mode I, II, and III SIFs along the crack front of an internal, inclined (at 45 degrees), penny-shaped crack under unit uniform uniaxial tension

4 Surface Ellipse Crack in a Plate – Raju-Newman Finite Element Solution

The handbook solution, for a surface crack in a finite plate, was developed by Raju and Newman (1979; 1986); the reported accuracy is within 5%. Typical plate dimensions are depicted in Fig 4.1. The loading consists of uniform unit traction and simple constraints.

An ANSYS FE model of the plate was created with dimensions: H=4, W=4, t=2. The model was imported into FRANC3D, and a crack, with dimensions: a=0.8 and c=0.8, was inserted, Fig 4.2. The default crack front template has a radius of 0.04 units, and 8 circumferential elements in 3 rings (quarter-point wedge and 2 rings of bricks) with an aspect ratio of 2. About 30,000 elements were generated for the entire model.

Based on the results from Section 2, we increase the number of rings in the template to 5 and set the aspect ratio to 1. We also turn off the Do Crack Mouth Coarsen Mesh flag in the meshing parameters dialog (see Chapter 7 of the User's Guide). About 123,000 elements are generated for the entire model with these template parameters.

A third, more refined, mesh is created by reducing the template radius and switching the volume meshing to ABAQUS. The template radius is 0.02 units, and 5 rings with an aspect ratio of 1 are used. This model contains about 242,000 elements.

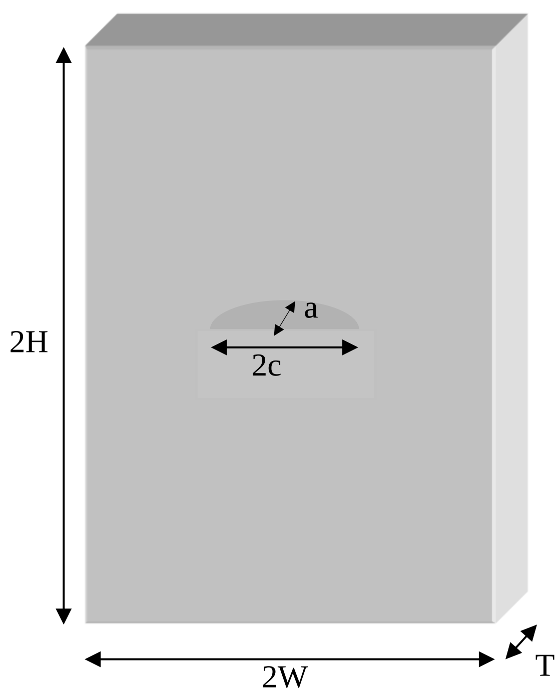


Figure 4.1 Raju/Newman surface crack in a plate model configuration.

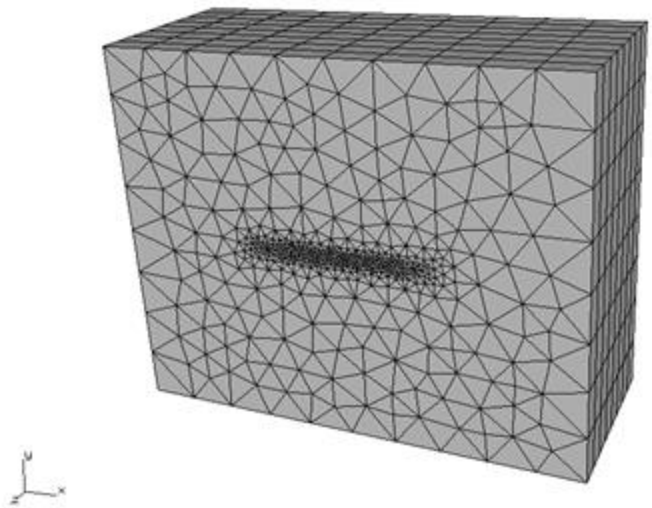


Figure 4.2 FRANC3D model of Raju-Newman surface crack in a plate.

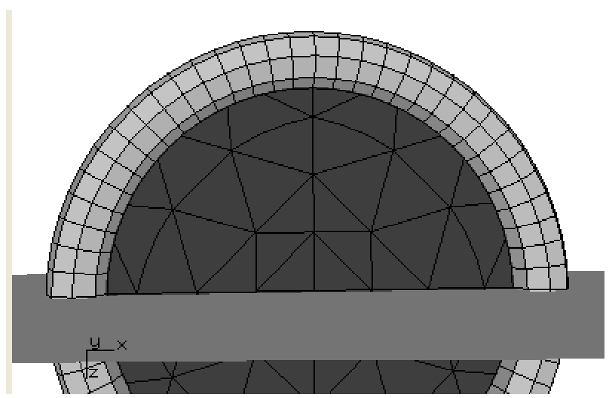


Figure 4.3 Default crack front template mesh.

The Mode I SIFs from FRANC3D are plotted in Fig 4.4 along with the Raju-Newman SIFs. There is minor difference in Mode I SIF results for the three meshes except at the free surface. For the first mesh, the maximum difference between the FRANC3D solution and the Raju-Newman solution is at the ends of the crack front, at the plate surface; the difference is about 2%. At the mid-point, the difference is about 0.5%. The refined mesh captures a drop in Mode I SIF at the surface, but the rest of the solution is nearly identical.

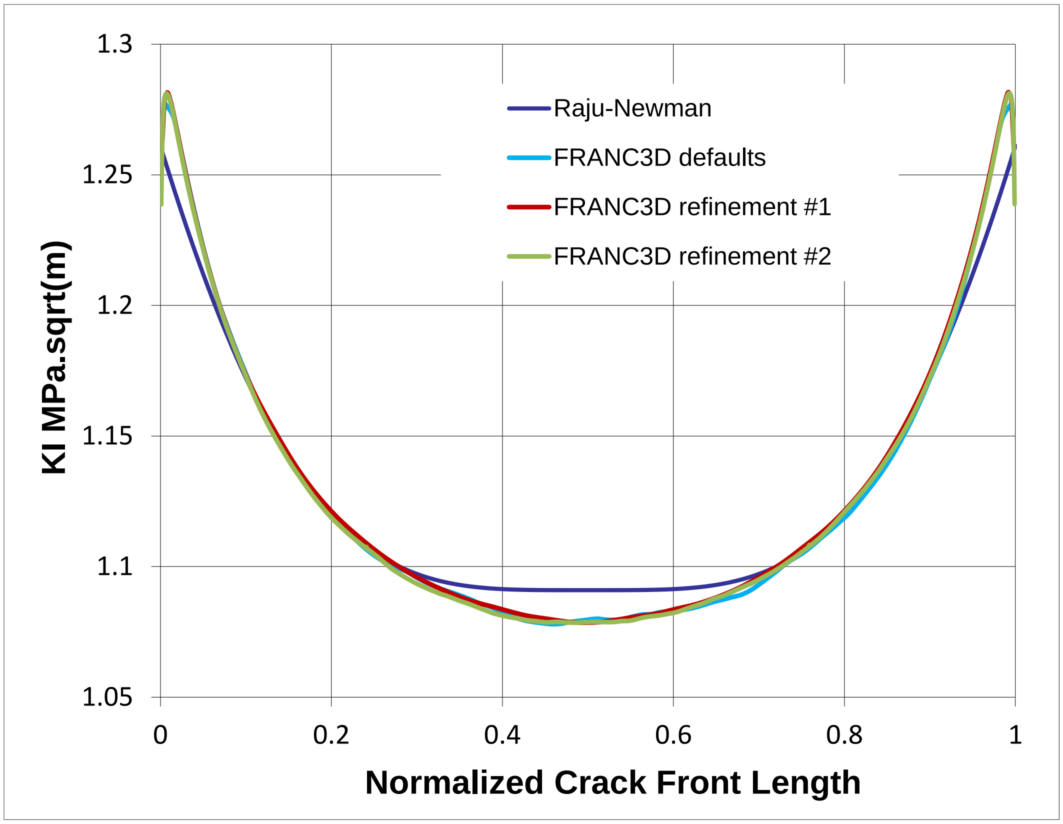


Figure 4.4 Raju-Newman surface crack in a plate – Mode I SIFs.

5 Through-Thickness Crack in a Plate

A through-thickness edge crack in a plate and a middle-through-thickness crack in a plate are standard benchmark problems. The model, Fig 5.1, is a 10x5x5 plate, and the boundary conditions consist of uniform traction and simple displacement constraints. The elastic modulus is 3.0e7 and Poisson's ratio is 0.30. The top and bottom surfaces of the plate have uniform unit traction.

Two sets of boundary conditions are applied to simulate a through-thickness edge crack and a middle-through-thickness crack. For the first case, only simple constraints are applied to prevent rigid body motion. For the second case, rollers in the x-direction are applied to the surface where the crack is inserted. A crack of 0.5 units is inserted on the left side, Fig 5.2.

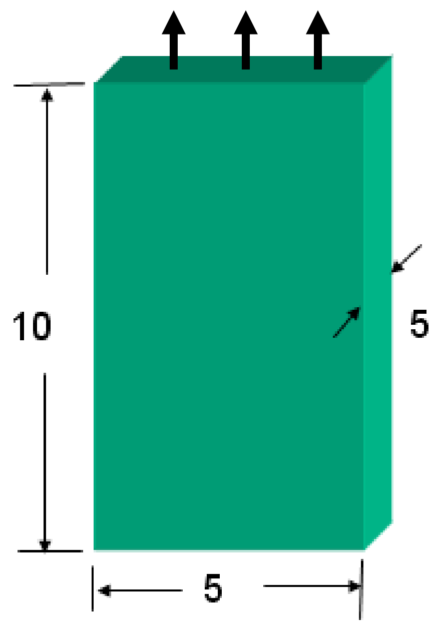


Figure 5.1 Plate model.

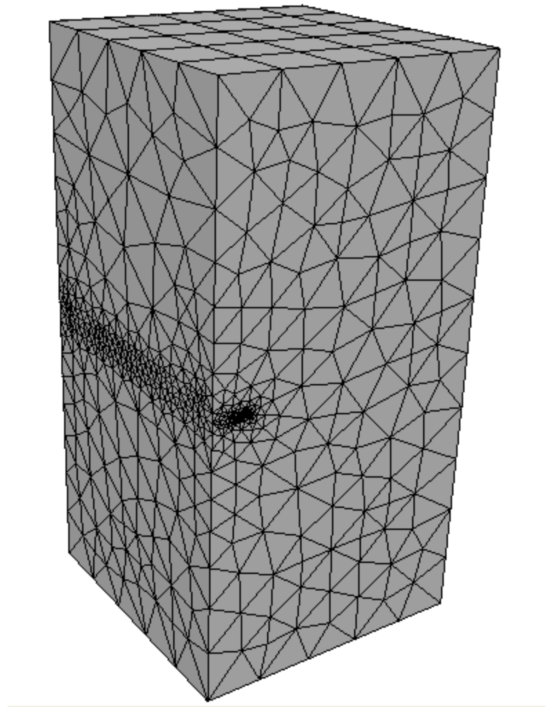


Figure 5.2 FRANC3D model with an edge crack of length 0.5 units.

5.1 Through-Thickness Edge Crack

The default crack front template mesh is shown in Fig 5.3. The default template radius is 0.05 units with 8 rings around the crack front and 3 rings of elements (quarter-point wedge and 2 rings of bricks). Based on the Section 4 results, we turn off the Do Crack Mouth Coarsen Mesh flag. To achieve a more refined mesh, we increase the number of element rings in the template to 5 and set the aspect ratio to 1.

These parameters are used for both the edge crack and the center crack (in the next section). About 55,000 elements are generated for the default parameters and 150,000 elements are generated for the more refined model.

Without the roller boundary conditions on the left face, the model represents a through-thickness edge crack. The handbook 2D solution is provided by the equation (Murakami, 1987):

$$K_I = \sigma\sqrt{\pi a} \quad F_I(\alpha)$$

The correction factor is:

$$\alpha = \frac{a}{w}$$

$$F_I(\alpha) = \left(1.12 - 0.231\alpha + 10.55\alpha^2 - 21.72\alpha^3 + 30.39\alpha^4\right)$$

for $a/w \leq 0.6$. $K_{II} = K_{III} = 0$.

This solution is insensitive to Poisson's ratio. In this model, a=0.5 and w=5, so the correction factor is 1.1837. Therefore, the handbook K_I value is 1.484. The FRANC3D SIFs, which are sensitive to the value of Poisson's ratio, are shown in Fig 5.4.

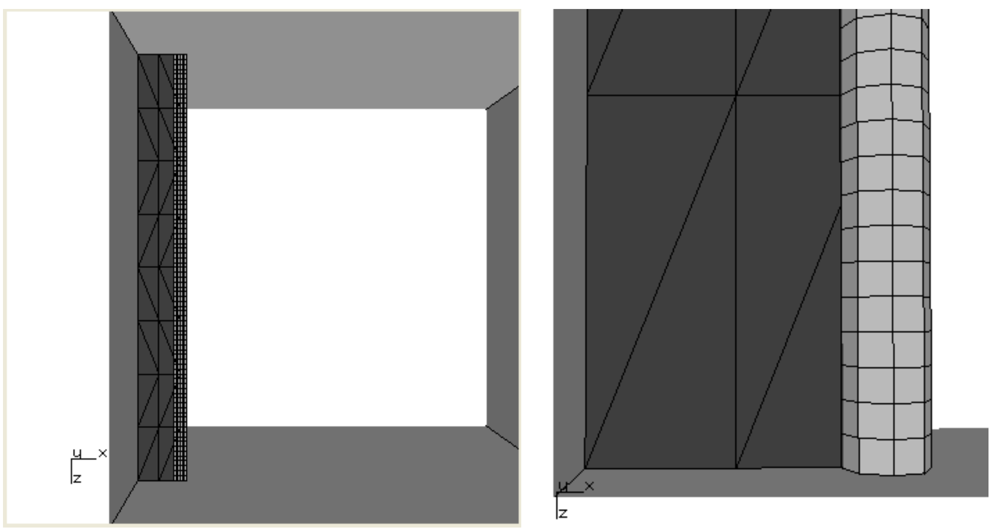


Figure 5.3 Crack front template elements for edge crack. The left panel shows the entire crack, and the right panel shows about 1/8th of the crack.

If we add constraint on the z-surfaces to simulate plane strain conditions, the maximum difference between the FRANC3D Mode I SIF value and the handbook value is 0.25%, and the average difference is 0.14%. The average Mode I SIF value for plane strain conditions is 1.485 compared to the handbook solution of 1.484.

The default template and meshing parameters produce comparable results, but there is significantly more numerical noise, Fig 5.5; the main reason is the variation in the size and shape of the tetrahedral elements near the crack front. It is much more apparent in this model, compared with the previous models, as the crack is shallow while the plate is relatively thick, which leads to a larger variation in element size and shape along the front.

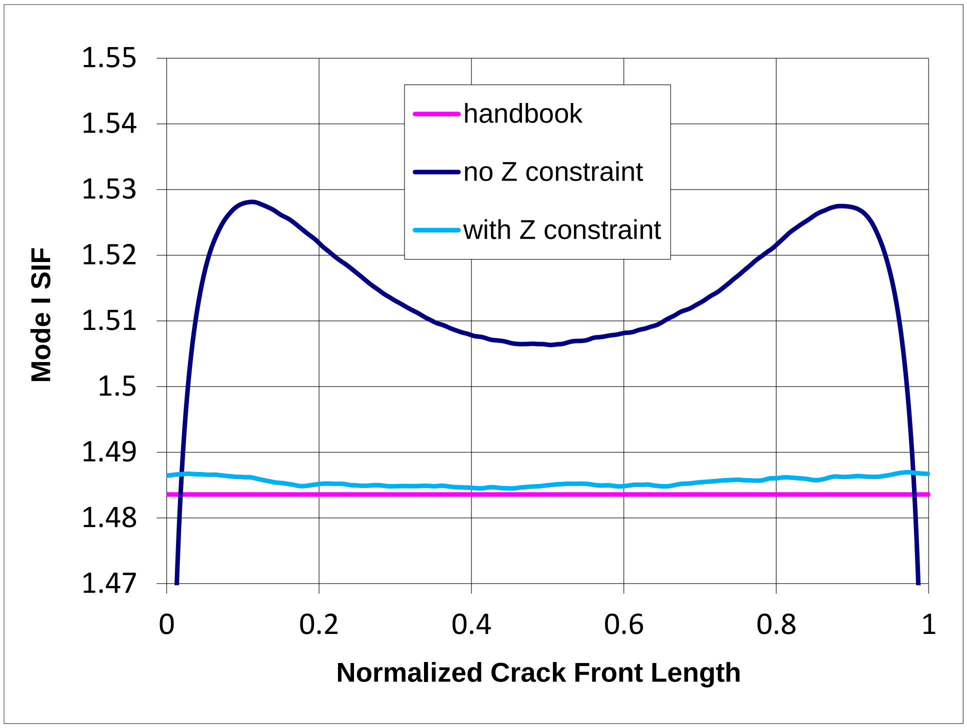


Figure 5.4 Mode I SIF values for a part-through edge crack compared to handbook.

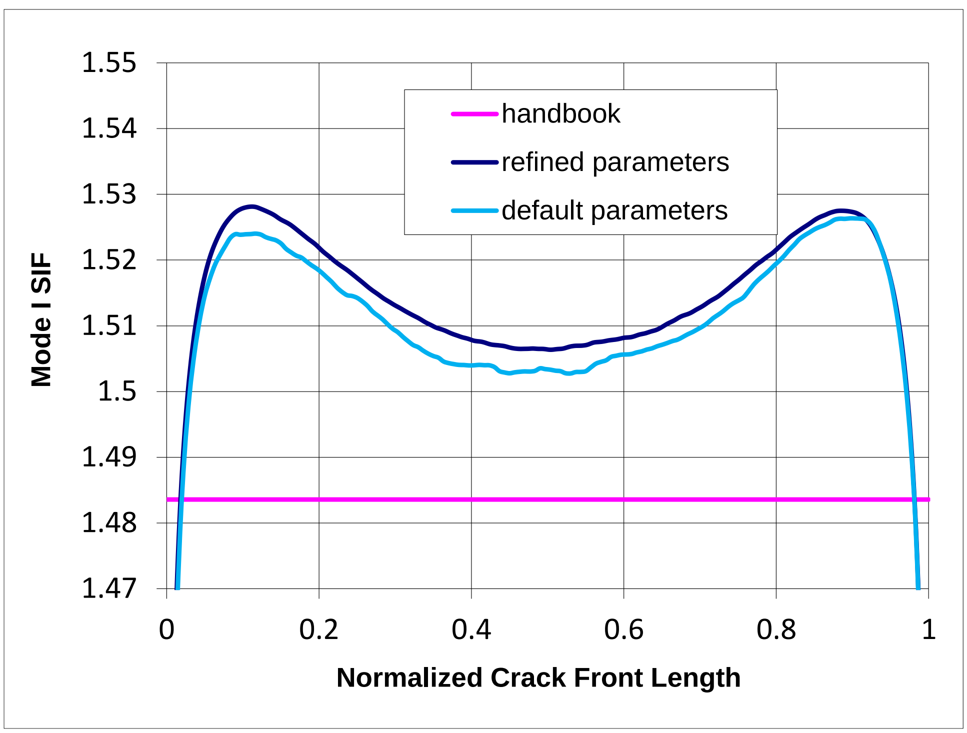


Figure 5.5 Mode I SIF values for a part-through edge crack with default and refined template mesh parameters.

5.2 Middle-Through-Thickness (Center) Crack

The handbook 2D solution for Mode I SIF for the center crack is given in Murakami (1987). The equations are:

$$K_I = \sigma\sqrt{\pi a} \quad F_I(\alpha, \beta)$$
$$\alpha = \frac{2a}{w}, \ \beta = \text{h/W}$$

The parameters h, W, and a are depicted in Fig 5.6. For this model, the correction factor, F_1, is 1.014, and the value of Mode I SIF is 1.271.

The FRANC3D Mode I SIF values are plotted in Fig 5.7. We used the ‘refined’ template mesh parameters. As with the edge crack, the FRANC3D model is neither plane strain nor plane stress. If we add constraints on the z-surfaces to simulate plane strain conditions, the average difference between the FRANC3D Mode I SIF value and the handbook value is 0.3%. The average Mode I SIF value for plane strain conditions is 1.267.

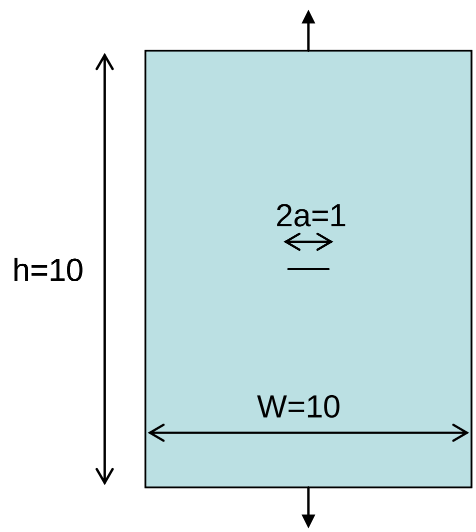


Figure 5.6 Center cracked plate under remote tension.

5.2.1 Symmetry Boundary Conditions

Symmetry boundary conditions are applied to the surface normal to the x-axis at x=0 in the uncracked FE model. When we import the FE model into FRANC3D, we retain the displacement boundary conditions without retaining the associated nodes and mesh facets. FRANC3D will transfer these constraints after crack insertion and remeshing. We could have simply doubled the plate geometry and redefined the crack geometry to create a model as shown in Fig 5.6, but symmetry reduces the model size.

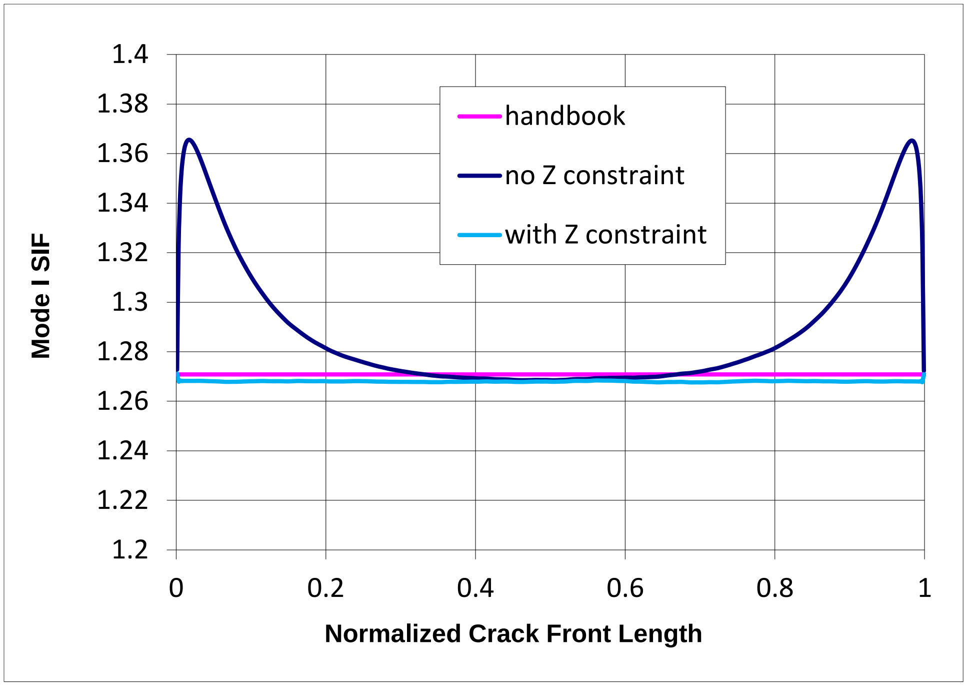


Figure 5.7 Mode I SIF values for middle-through crack.

When we write the cracked FE model file, all transferred boundary conditions are included. The symmetry boundary conditions on the surface where the crack is inserted can verified using the FE pre-processor. For example, Fig 5.8 shows the ABAQUS version of the cracked symmetry model (without z-constraint).

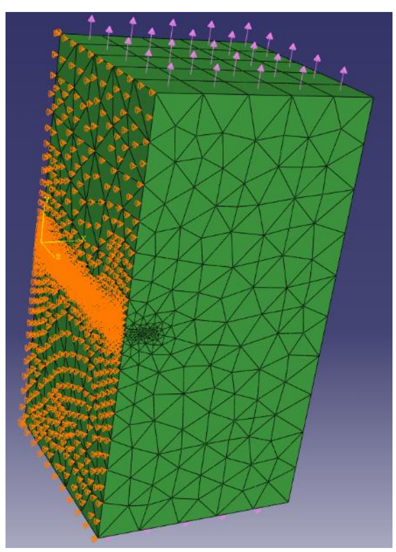


Figure 5.8 Boundary conditions shown in ABAQUS CAE for middle-crack model.

6 Thick Plate with Middle Through Crack and Anisotropic Material Properties

A 'thick plate' model has been used to verify that FRANC3D produces accurate stress intensity factors using the M-Integral for isotropic and anisotropic material properties (Banks-Sills *et al.*, 2007).

Two different sets of boundary conditions are used with this model: 1) simple uniform tension and 2) simple shear. The outer portion of the model is shown in Fig 6.1 with the mesh retained for the boundary conditions and for the cut-surfaces. The inner portion of the model is shown in Fig 6.2 with a crack inserted.

The original plate is 30x30x15 units and the through crack is 2 units wide; plane strain conditions are approximated along the crack front in the middle of the plate. The plate is first analyzed with uniform tension and simple supports, Fig 6.3. The material properties are provided in Table 1.

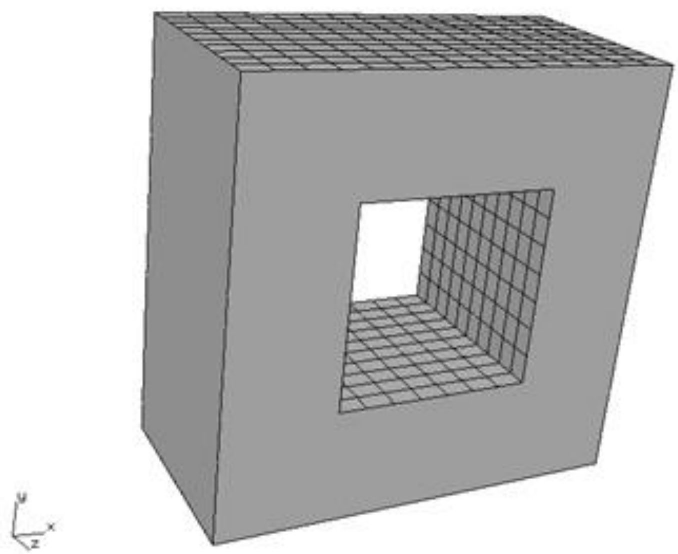


Figure 6.1: Outer portion of the thick plate model with tension boundary conditions.

Table 1: Material Properties

	E_x, E_y, E_z	Nu_{xy}, Nu_{yz}, Nu_{xz}	G_{xy}, G_{yz}, G_{xz}
isotropic	950, 950, 950	0.3, 0.3, 0.3	
orthotropic	950, 950, 2400	0.45, 0.3, 0.3	328, 231, 231

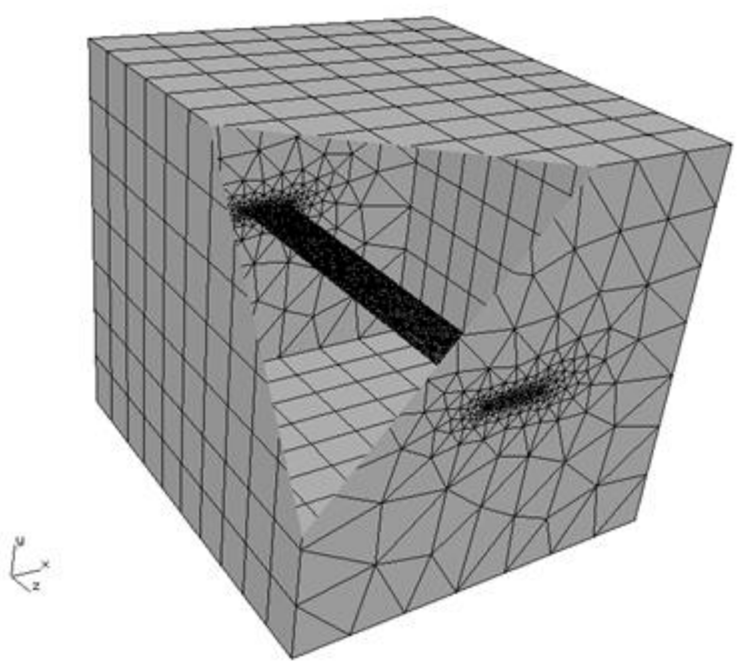


Figure 6.2 Thick plate inner portion with a middle through crack.

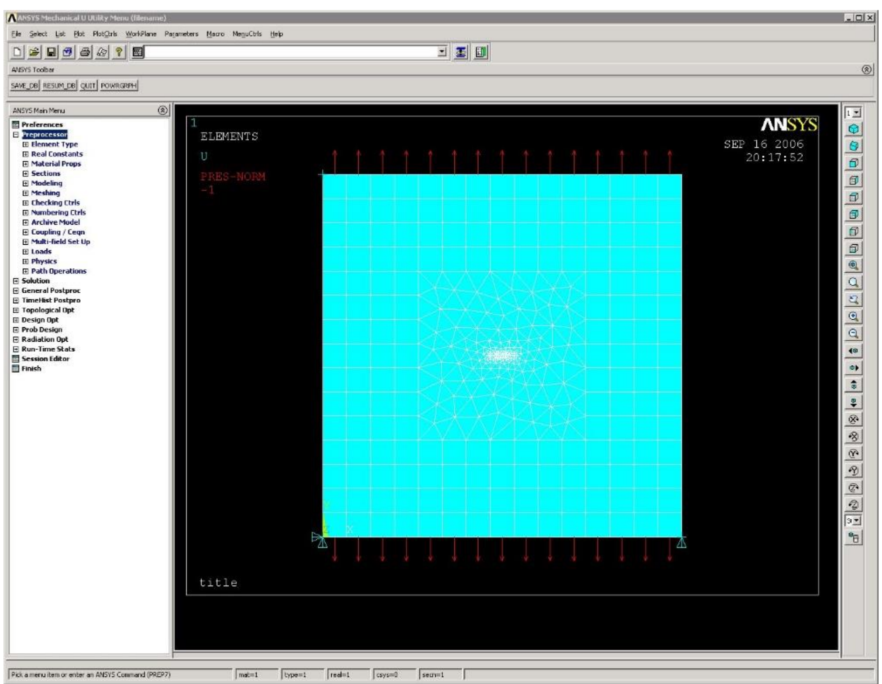


Figure 6.3 Thick plate with a middle through crack; tension boundary conditions and simple constraints in ANSYS.

6.1 Uniform Tension

The stress analyses are performed using ANSYS. The deformed shape, with maximum principal stress contours, is shown in Fig 6.4.

The template radius is 0.1 for both crack fronts, with 8 rings around the crack front and 5 rings of elements. ABAQUS volume meshing is used, and we turn off the Do Crack Mouth Coarsen Mesh flag.

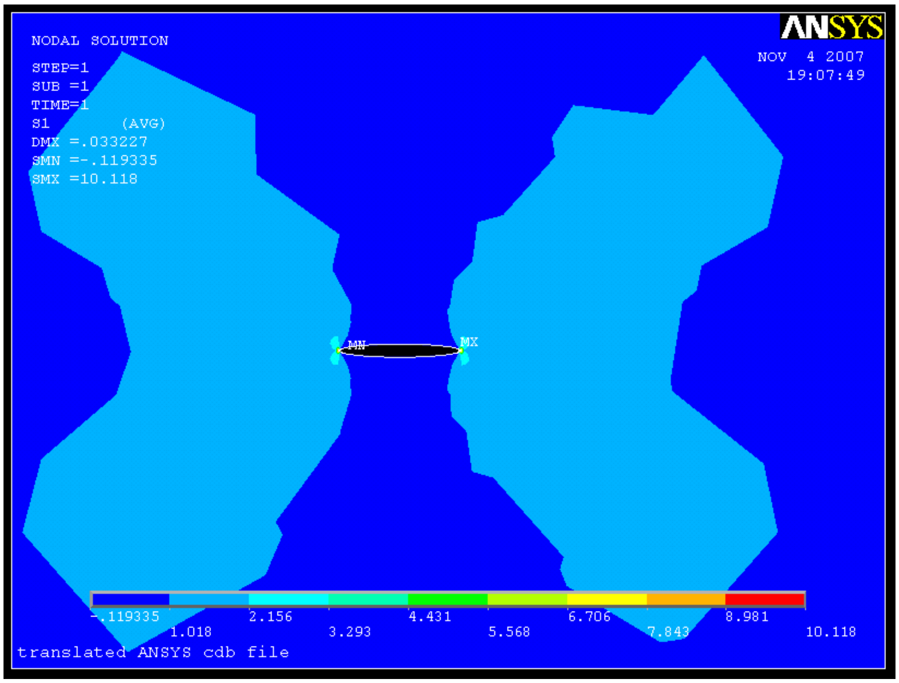


Figure 6.4 Thick plate deformed crack with maximum principal stress contours.

The analytical plane strain solution for the Mode I SIFs for the isotropic case under uniform tension is defined as: $K_I = \sigma\sqrt{\pi a}$ where σ=1 and a=1, resulting in K_I=1.772. The SIFs are plotted in Fig 6.5.

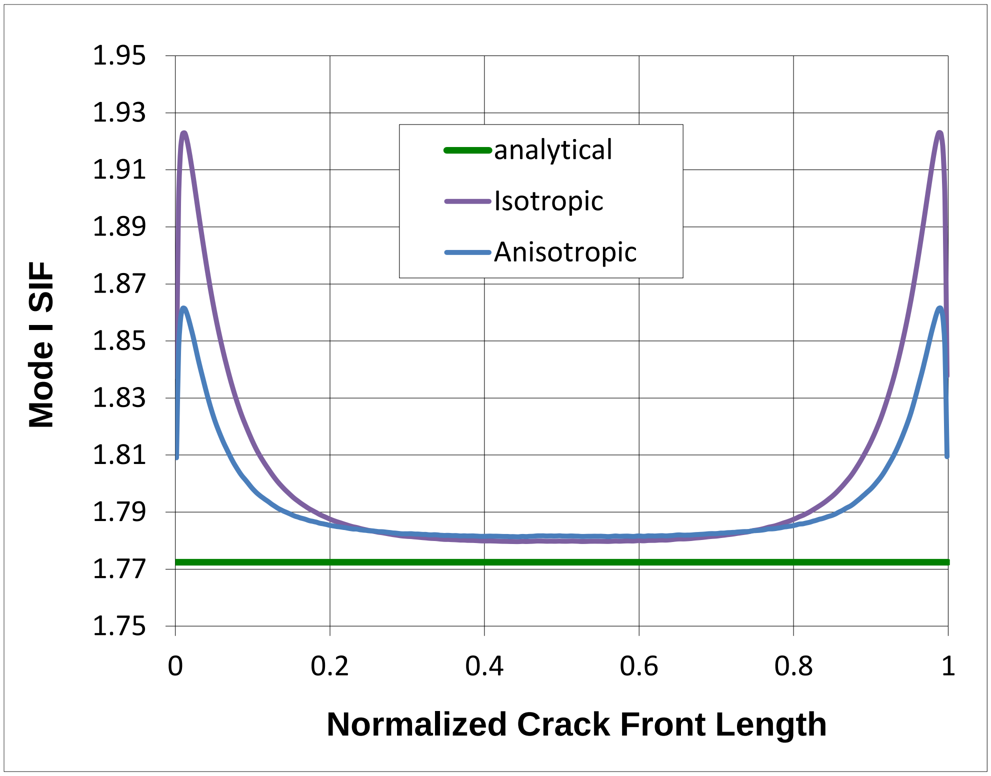


Figure 6.5 Mode I SIFs for thick plate under uniform tension for isotropic and anisotropic material properties.

6.2 Pure Shear

The second set of boundary conditions represents pure shear; Fig 6.6 shows the deformed shape from ANSYS of the original uncracked plate. The same crack front template and meshing parameters from Section 6.1 are used. The Mode II SIFs for pure shear are plotted in Fig 6.7; Mode I SIFs are zero.

Note that the application of pure shear in ANSYS requires the use of surface effect elements and the shear is applied in the element coordinate system. As for the tension case, the outer portion of the model was retained as a global model (see Fig 6.1), and the inner portion was extracted for crack insertion and remeshing. When the surface effect elements and shear tractions remain with the global model portion, FRANC3D can pass the elements and loads through to the cracked model.

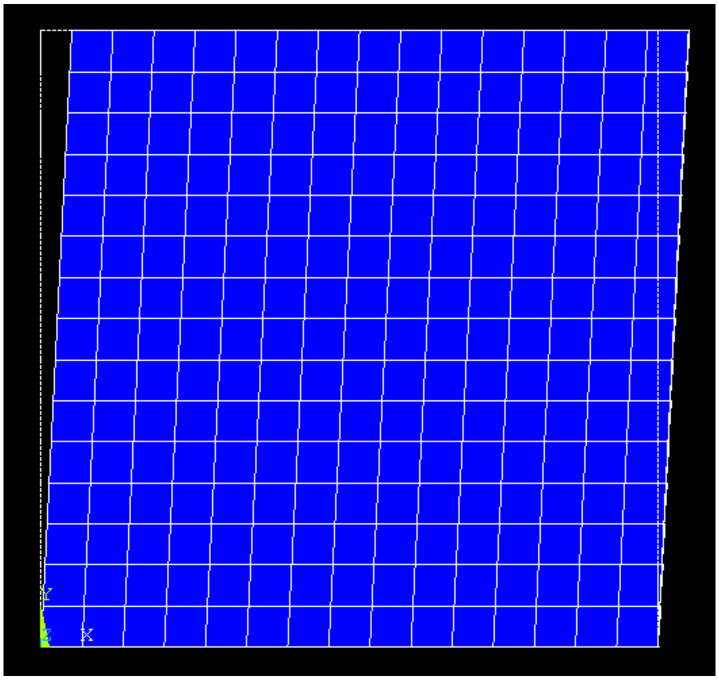


Figure 6.6 Deformed shape under pure shear boundary conditions.

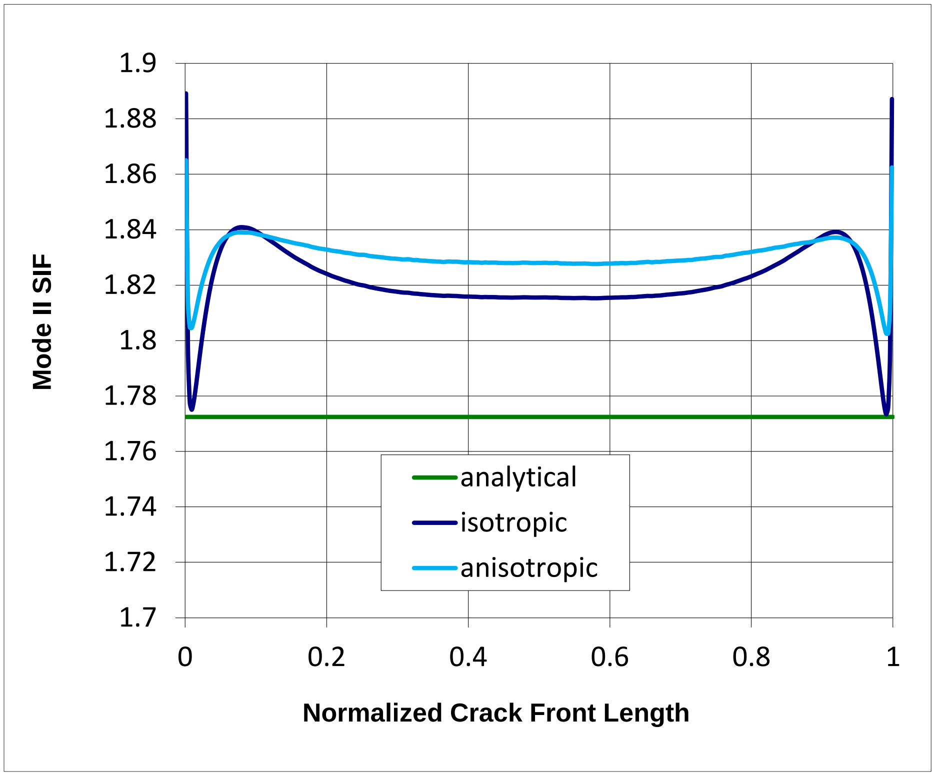


Figure 6.7 SIFs for pure shear for isotropic and anisotropic material properties.

7 Corner Crack in a Plate with a Hole

A plate with a corner cracked hole is another standard benchmark problem. The uncracked plate is created using ANSYS and is 40x40x1 units with a hole of radius 1 unit, Fig 7.1.

Note that only half of the plate is modeled, and symmetry boundary conditions are used, which implies that there will be two symmetric cracks emanating from the hole; this is consistent with the handbook solution. The elastic modulus is set to 10,000 and Poisson's ratio is 0.3. The boundary conditions consist of uniform tension and simple displacement constraints.

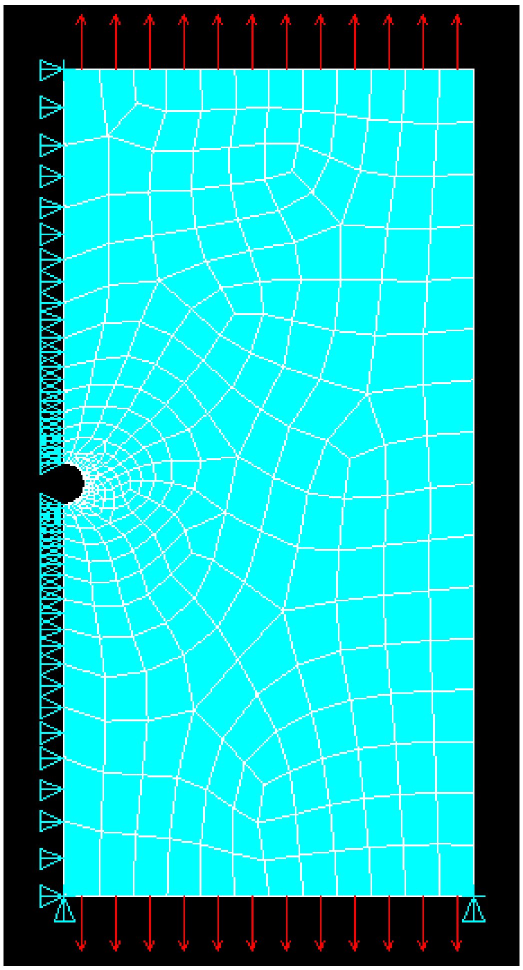


Figure 7.1 Plate with hole under uniform tension in ANSYS.

A quarter-circular corner crack is inserted at the edge of the hole; the radius of the crack is 0.05 units. The default crack front template mesh is shown in Fig 7.2; the template radius is 0.005 units. Based on the previous models, we switch to ABAQUS volume meshing, turn off the Do Coarsen Crack Mouth Mesh flag and increase the number of rings in the template to 5.

About 32,000 elements are generated. The subsequent analysis is done using ANSYS; the deformed shape and the maximum principal stress contours near the corner crack are shown in Fig 7.3.

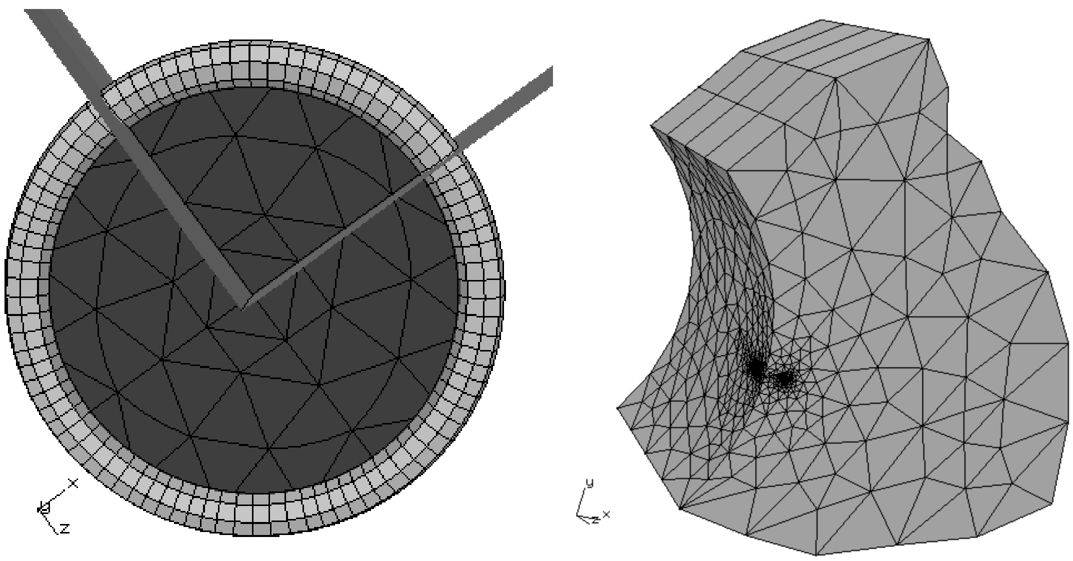


Figure 7.2 Plate with hole and corner crack showing the front template mesh and final surface mesh.

The stress is higher on the inside of the hole than on the surface of the plate, and the Mode I SIF reflects this, Fig 7.4. The handbook numerical solution is available in Murakami (1987) and from Newman and Raju (1986). The equations from Newman and Raju (1986) have been encoded in Excel and the handbook values are plotted in Fig 7.4 also. The difference between the computed and handbook values is significant (~10%).

Additional reference solutions (Shin, 1990; Lin and Smith, 1999) indicate similar discrepancies with the Newman and Raju (1986) solution for this model.

The Mode I SIF curves for Nu=0.0 and for Nu=0.45 are shown in Fig 7.5. For Nu=0.0, most of the FRANC3D curve matches the handbook solution, but the values at both ends are significantly higher than the handbook values.

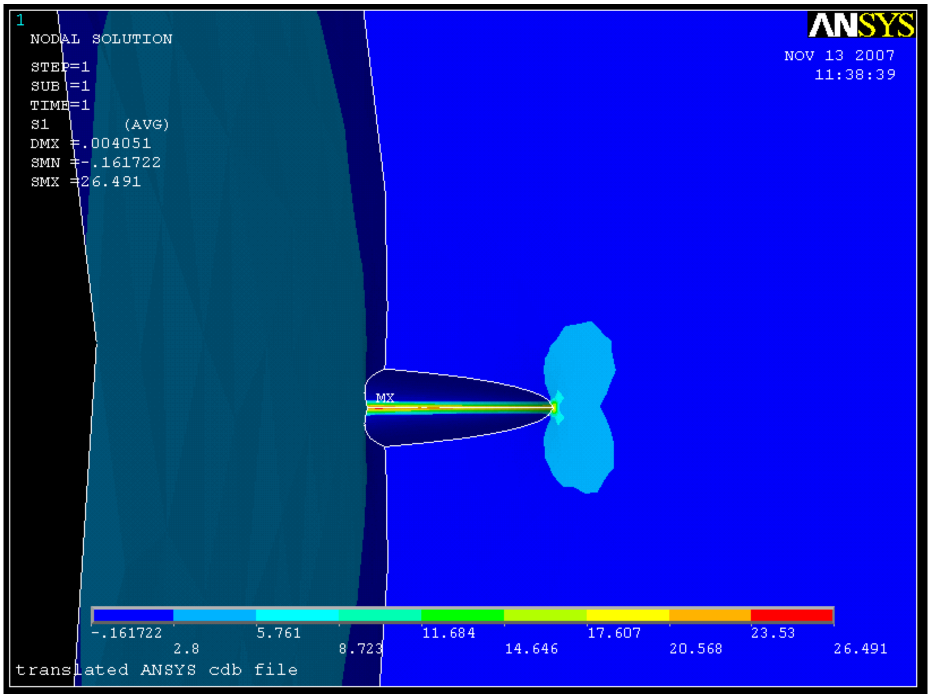


Figure 7.3 Plate with hole and corner crack - deformed shape and maximum principal stress contours.

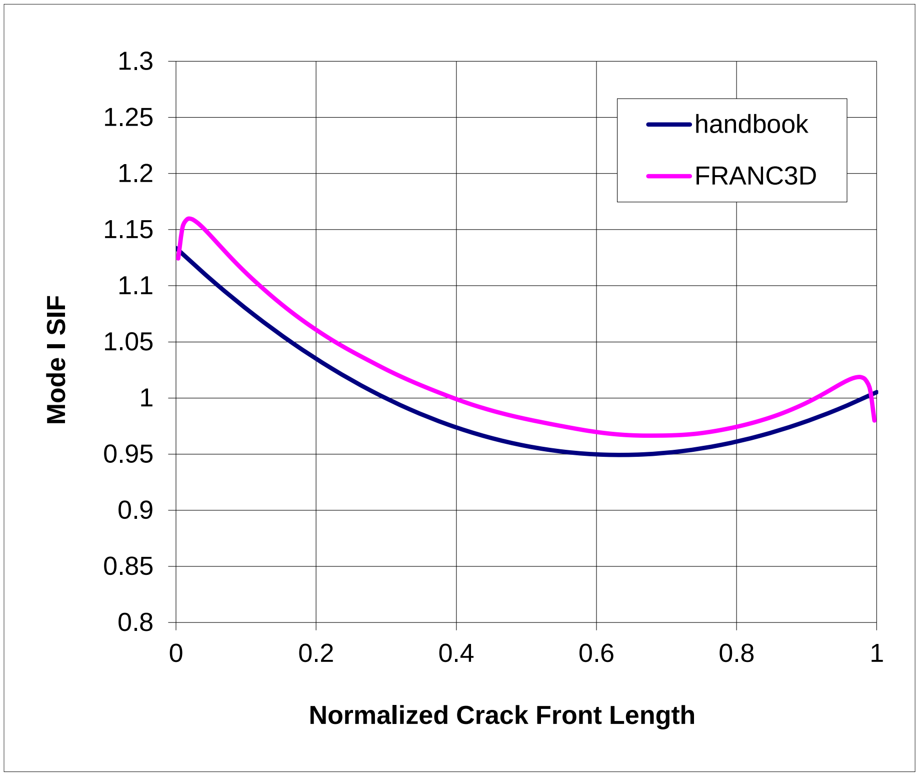


Figure 7.4 Mode I SIF for corner crack in plate with hole. Position 0 is on the surface inside the hole and position 1 is on the front plate surface.

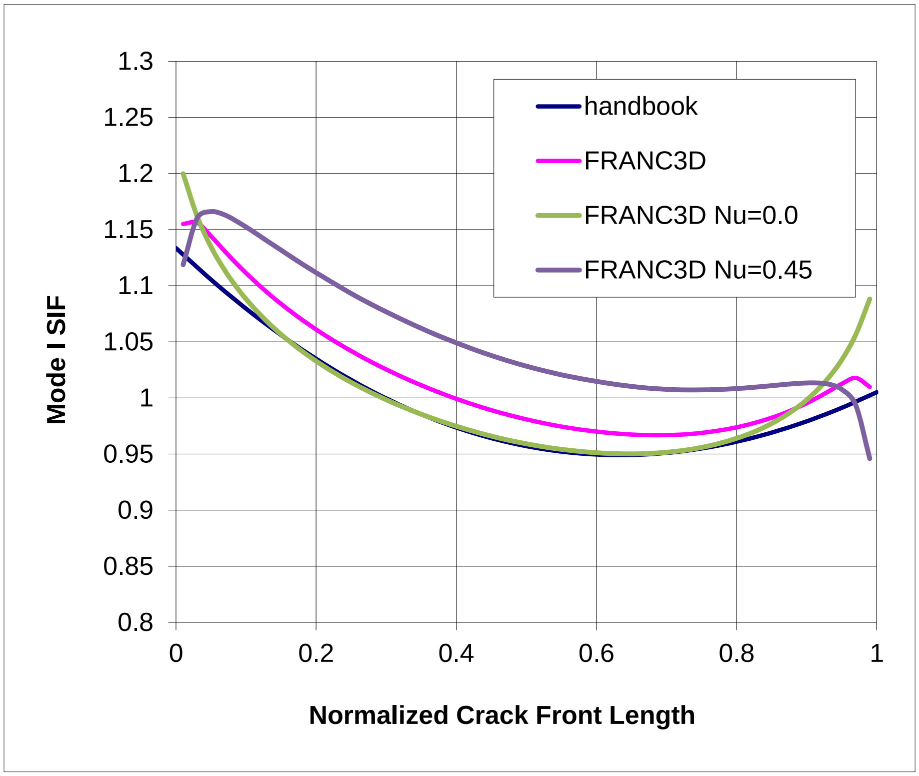


Figure 7.5 Mode I SIF for corner crack in plate with hole for different Nu values.

8 Compact Tension Specimen

A compact tension specimen is created in ABAQUS, Fig 8.1. A sketch with relevant dimensions and loads is shown in Fig 8.2. The 2D plane strain equation for K_I is shown below the sketch. The first equation is based on 1970 work, while the second equation is from Bower (2009) and is considered more accurate.

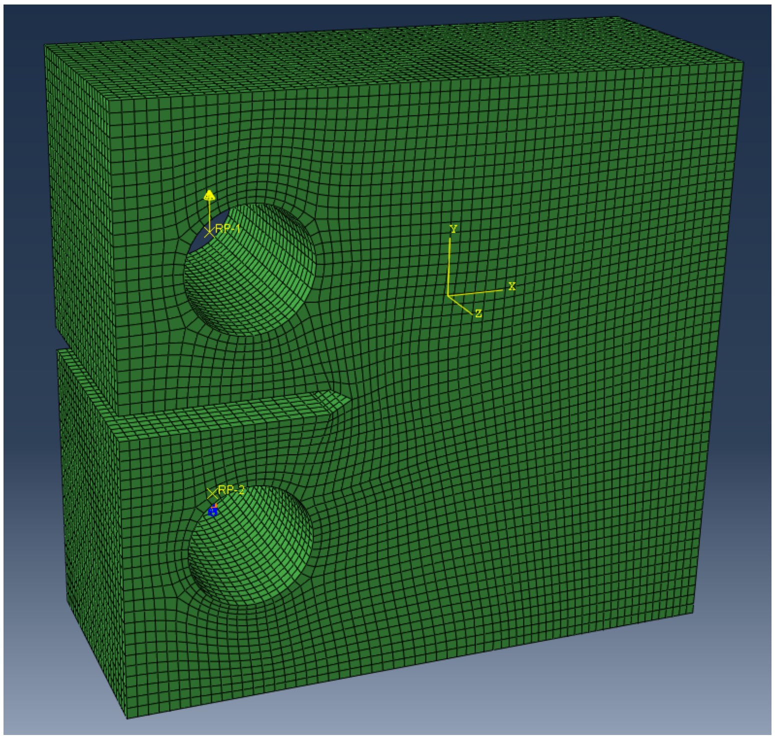
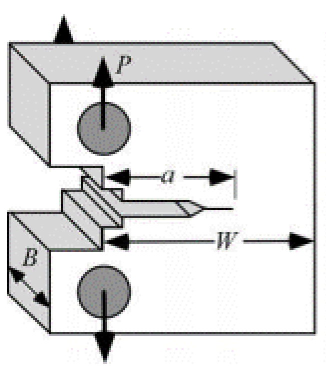


Figure 8.1 Compact tension specimen in ABAQUS.

A crack is inserted into a local portion of the FE model, which makes the dimension 'a' equal to 20 mm. W is 50.4 mm and B is 25.4 mm. The load P is 26358 N. The material properties are set to 20500 and 0.33 for E and Nu.



$$K_I = \frac{\text{LOAD}}{BW^{\frac{1}{2}}} \cdot f\left(\frac{a}{W}\right),$$

where

$$f\left(\frac{a}{W}\right) = \frac{\left(2+\frac{a}{W}\right)\left\{0.886+4.64\left(\frac{a}{W}\right)-13.32\left(\frac{a}{W}\right)^2+14.72\left(\frac{a}{W}\right)^3-5.6\left(\frac{a}{W}\right)^4\right\}}{\left(1-\frac{a}{W}\right)^{\frac{3}{2}}}$$

$$K_I = \frac{P}{B}\sqrt{\frac{\pi}{W}}\left[16.7\left(\frac{a}{W}\right)^{1/2} - 104.7\left(\frac{a}{W}\right)^{3/2} + 369.9\left(\frac{a}{W}\right)^{5/2} - 573.8\left(\frac{a}{W}\right)^{7/2} + 360.5\left(\frac{a}{W}\right)^{9/2}\right]$$

Figure 8.2 Compact tension SIF equations.

The FRANC3D values for K_I are shown in Figure 8.3. As in Section 5, it is clear that the model is not in plane strain or plane stress. To simulate the conditions of plane strain, we set the Poisson's ratio to 0.0 and redo the analysis. The result is a 'constant' value for K_I along the crack front, Fig 8.4. The average value is 1063.6, and the second equation in Fig 8.2 gives a value of 1058.3 MPa√mm.

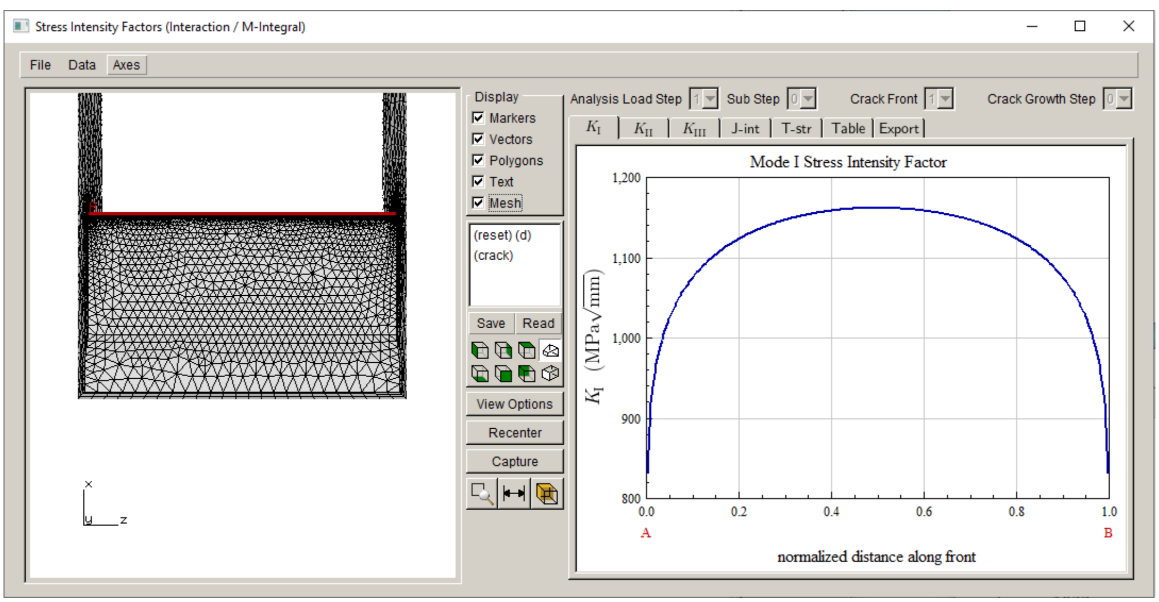


Figure 8.3 Compact tension Mode I SIF with Nu=0.33.

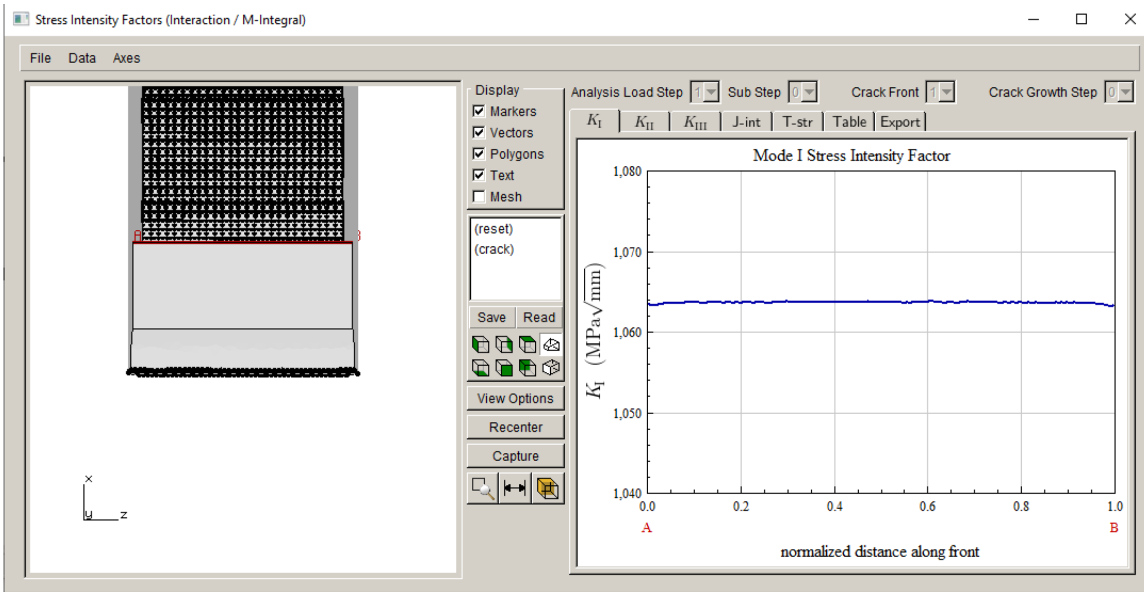


Figure 8.4 Compact tension Mode I SIF with Nu=0.

Rather than setting the Poisson's ratio to zero, z-constraints could be added to the 'front' and 'back' surfaces of the model to simulate plane strain, as was done in Section 5. The resulting SIFs should be 'constant' and match the above SIFs; this is left as an exercise for the reader.

9 Summary

The benchmark model files are available for download from the FAC website.

Additional benchmark models can be found in Murakami (1987) or in fracture mechanics texts, such as Anderson (1991).

We have not provided any benchmarks for fatigue life (cycle) counts. Tutorial #6 (see the Tutorials 2-14 document) describes a typical fatigue test specimen and the corresponding FRANC3D simulation; that example is intended to simply be a tutorial to describe how one could benchmark the fatigue life computations.

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